Economics 2301

Lecture 2

Limits

Limits and Continuity

- It is often necessary to evaluate a function as its argument approaches some value.
- The **limit** of a function as its argument approaches some number *a* is simply the number the function’s value approaches as the argument approaches *a* , either from smaller values of *a*, giving the **left-hand limit**, or from larger values of *a*, giving the **right-hand limit**.

Left-hand Limit

The left-hand limit of a function $f(\mathrm{x})$ as its argument approaches some number a, written as

$\lim\limits_{x \to a^-} f(x)$ exists and is equal to L^L if, for any arbitrarily small number ε, there exists a small number δ such that

$$| f(x) - L^L | < \varepsilon$$

whenever $a - \delta < x < a.$

Right-hand Limit

The right-hand limit of a function $f(x)$ as its argument approaches some number a, written as

$\lim_{x \to a+} f(x)$ exists and is equal to L^R if, for any arbitrarily small number ε, there exists a small number δ such that

$$|f(x) - L^R| < \varepsilon$$

whenever $a < x < a + \delta$.

Limit

When the left - hand limit equals the right - hand limit, we can simplify the notation by suppressing the superscripts and defining

$$\lim_{x \to a} f(x) = \lim_{x \to a^-} f(x) = \lim_{x \to a+} f(x)$$

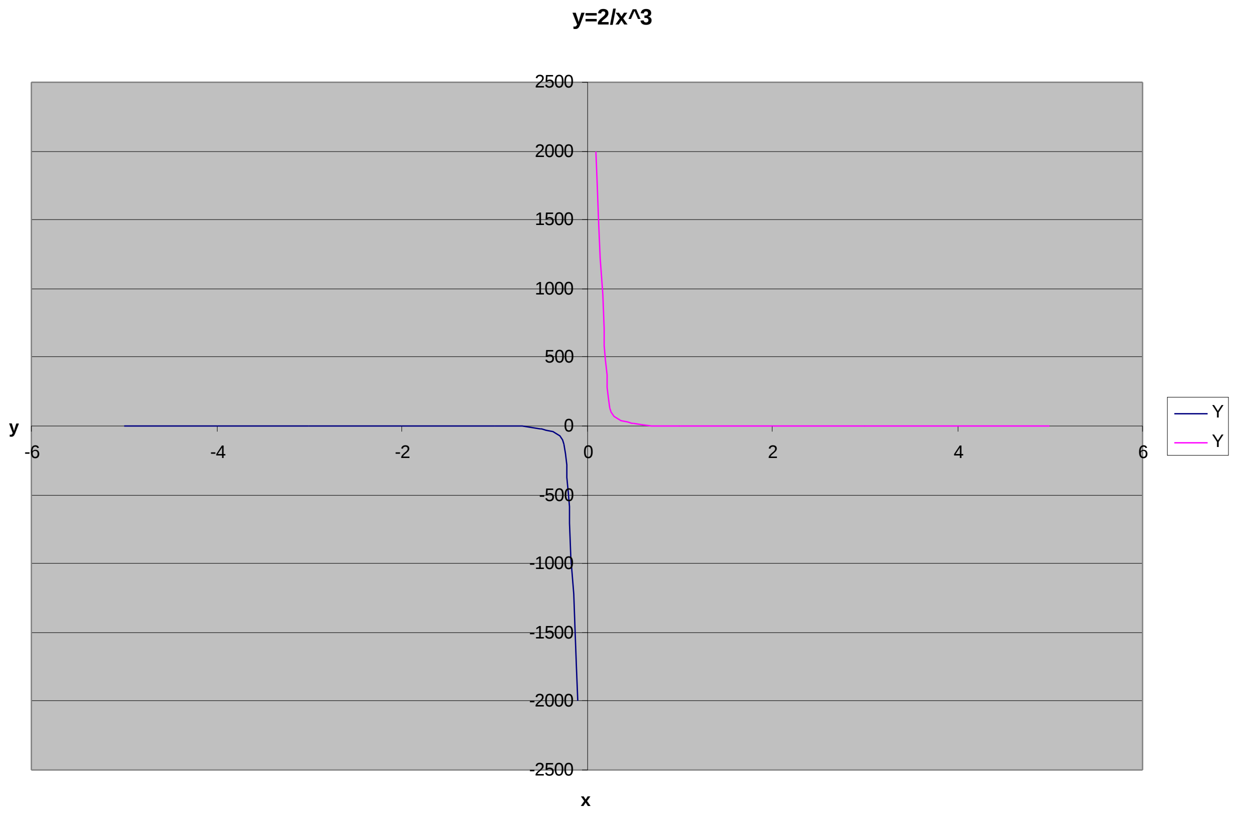
Infinite Limits

The limit of a function as its argument approaches *a* equals positive infinity if the value of the function increases without bound, and equals negative infinity if the value decreases without bound.

$\lim_{x \to a} f(x) = +\infty$ if, for every N > 0, there exits a $\delta > 0$ so that $f(x) > N$ whenever $a - \delta < x < a + \delta$.

Also, we have $\lim_{x \to a} f(x) = -\infty$ if, for every N < 0, there is a $\delta > 0$ so that $f(x) < N$ whenever $a - \delta < x < a + \delta$.

Left-hand limit ≠ Right-hand limit



Rules for Evaluating Limits

$Rule\ 1$

$$\lim_{x\to 0^-} m(k+x) = \lim_{x\to 0^+} m(k+x) = mk$$

$Rule\ 2$

$$\lim_{x\to\infty} \frac{k}{(m\cdot x)+h} = 0$$

Example 1 of Limit

$$\lim_{x \to 0^+} \left(\frac{x^2 - 9}{x + 3} \right) = \lim_{x \to 0^+} (x - 3) = -3$$

Example 2 of Limit

$$\lim_{x\to\infty}\left(\frac{10}{5x}+20\right)=20$$

Our previous function

$$\lim_{x \to 0^+} \left(\frac{2}{x^3} \right) = \infty$$

$$\lim_{x \to 0^-} \left(\frac{2}{x^3} \right) = -\infty$$

Continuity

A continuous univariate function has no "breaks" or " jumps."

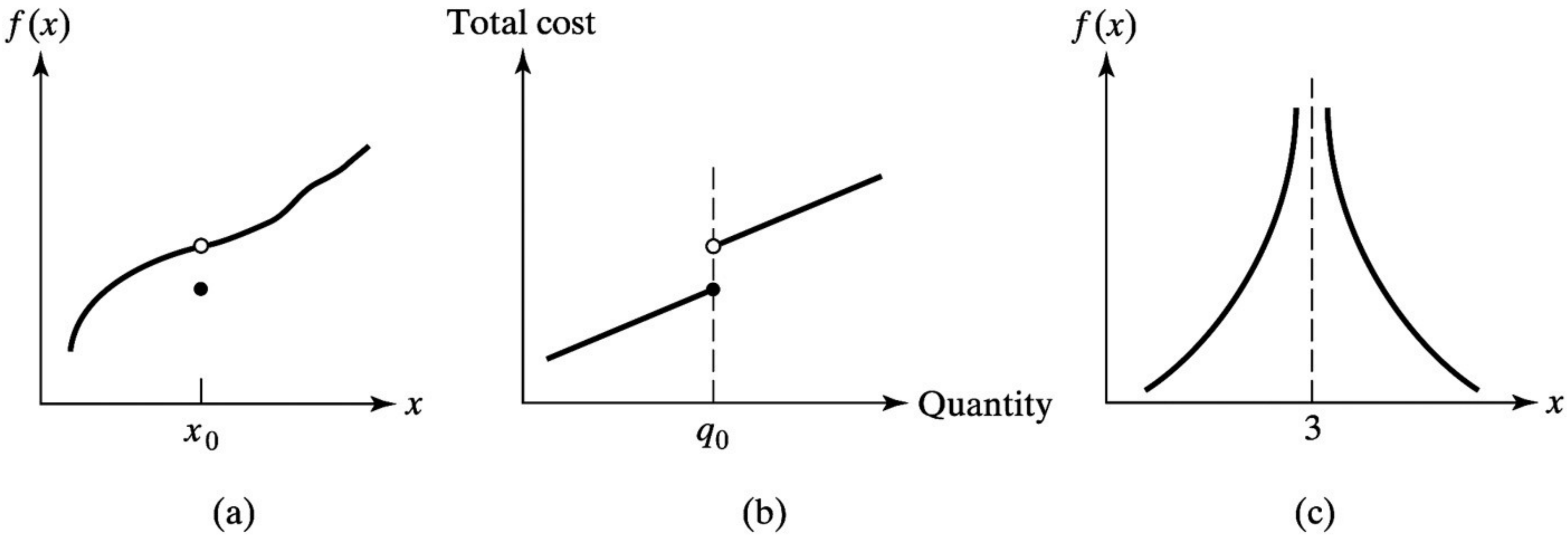
A function $f(x)$ is continous at $x = a$, where is in the domain of f, if the left - and right - hand limits at $x = a$ exist and are equal.

$$\operatorname{Lim}_{x\to a} f(x) = \operatorname{Lim}_{x\to a^+} f(x) = \operatorname{Lim}_{x\to a^-} f(x)$$

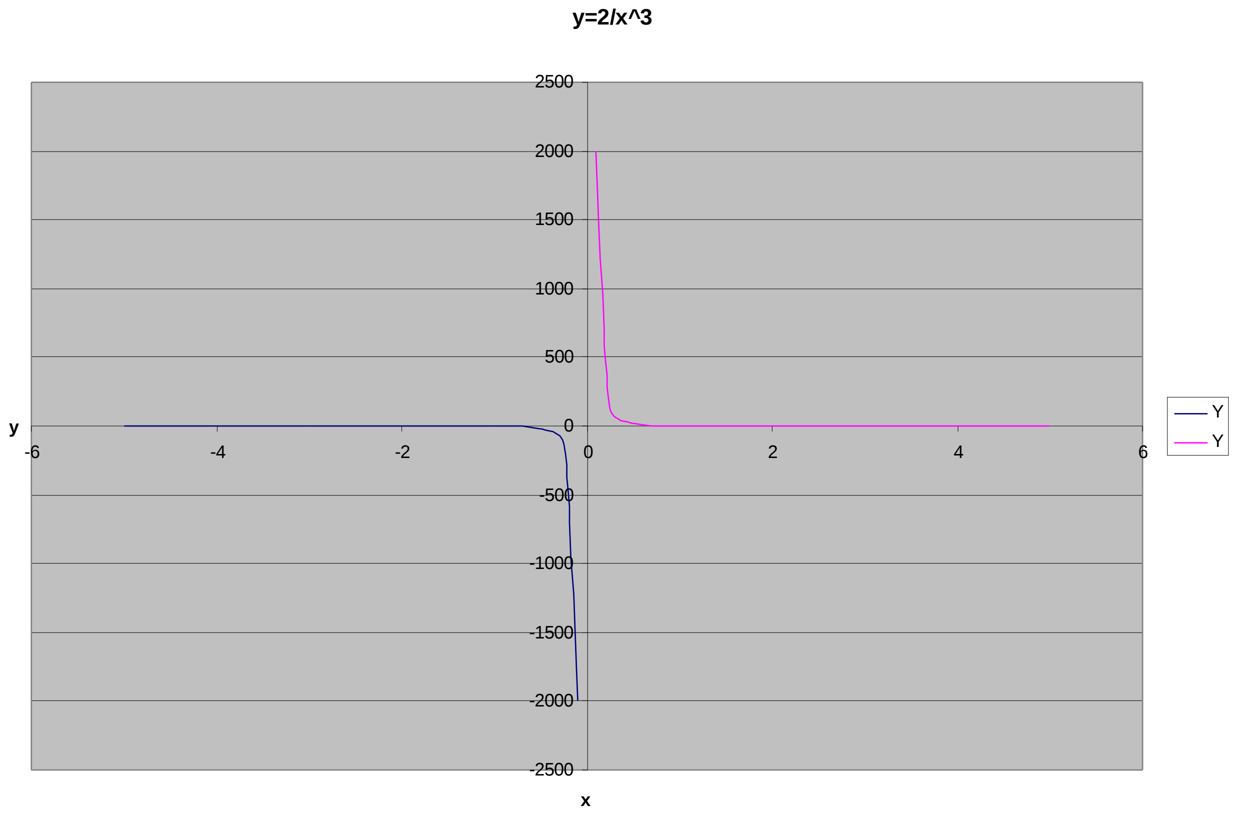
and the limit as $x \to a$ equals the value of the function at that point,

$$\operatorname{Lim}_{x\to a} f(x) = f(a)$$

Figure 2.5 Functions That Are Not Continuous



Our function that was not continuous



Our noncontinuous function

We know from a previous slide that

$$\lim_{x \to 0^+} \left(\frac{2}{x^3} \right) = \infty \quad \textit{and} \quad \lim_{x \to 0^-} \left(\frac{2}{x^3} \right) = -\infty$$

$f(x) = \dfrac{2}{x^3}$ at $x = 0$ *is* $\dfrac{2}{0}$ which is not defined.

Vertical asymptote

A vertical asymptote of a function occurs at a point when either a left-hand limit or a right-hand limit approaches positive infinity or negative infinity at that point. A function is discontinuous at a point where there is a vertical asymptote.