Economics 2301

Lecture 3

Properties of Functions

Increasing & Decreasing Functions

A function $y = f(x)$ is increasing, strictly increasing, decreasing, or strictly decreaing if it meets the following critera for any two of its arguments, x_A and x_B, where $x_B > x_A$.

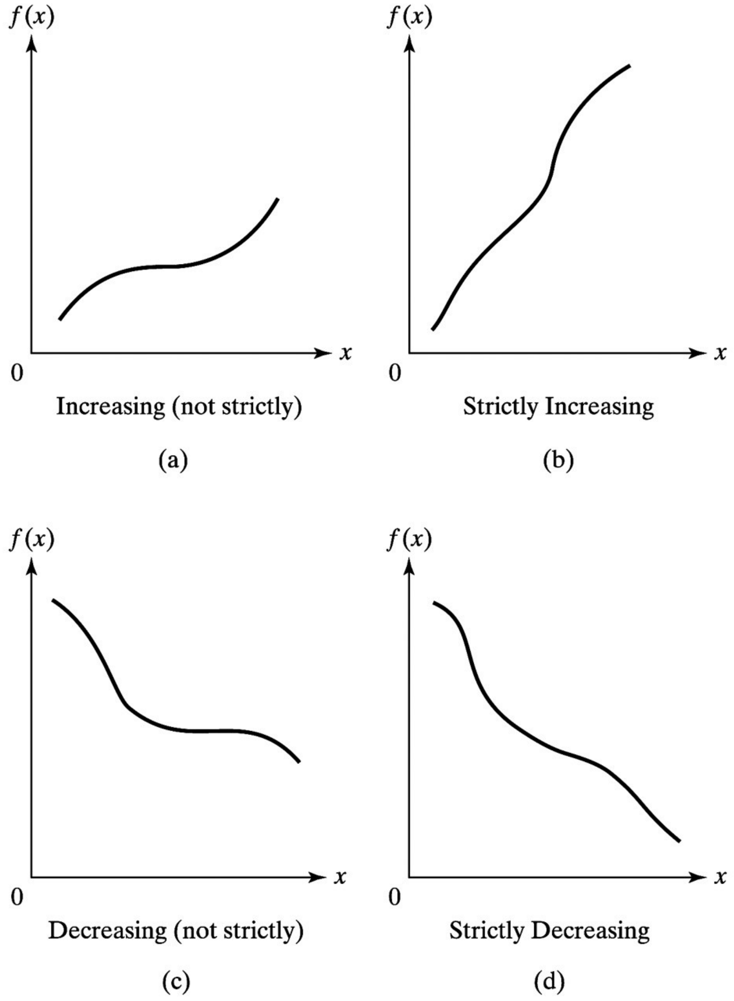
A function is increasing if $f\left(x_B\right) \geq f\left(x_A\right)$.

A function is strictly increasing if $f\left(x_B\right) > f\left(x_A\right)$.

A function is decreasing if $f\left(x_B\right) \leq f\left(x_A\right)$.

A function is strictly decreasing if $f\left(x_B\right) < f\left(x_A\right)$.

Figure 2.6 Increasing Functions and Decreasing Functions



Example functions

$$\text{Increasing function}: y = \text{int}\left(x\right),\ x > 0$$

$$\text{Strictly increasing function}: y = 2 + 2x$$

$$\text{Decreasing function}: y = \frac{2}{1 + \text{int}\left(x\right)} \quad x > 0$$

$$\text{Strictly decreasing function}: y = \frac{2}{1 + x} \quad x > 0$$

Monotonic Functions

- **Monotonic** if it increasing or if it is decreasing.
- **Strictly monotonic** if it is strictly increasing or if it is strictly decreasing.
- **Non-monotonic** if it is strictly increasing over some interval and strictly decreasing over another interval.
- Strictly monotonic functions are **one-to-one functions.**

One-to-One Function

- A function *f*(x) is one-to-one if for any two values of the argument x_1 and x_2, $f(x_1) = f(x_2)$ implies $x_1 = x_2$.
- An one-to-one function ha an **inverse function**.
- The inverse of the function $y=f(x)$ is written $y=f^{-1}(x)$.

Inverse Function

We find the inverse of a function $y = f(x)$ by solving for x in terms of y and then interchanging x and y to obtain $y = f^{-1}(x)$.

Note, some books do not make the interchange and hence, $\mathrm{x} = f^{-1}(f(x)) = f^{-1}(y)$.

Example

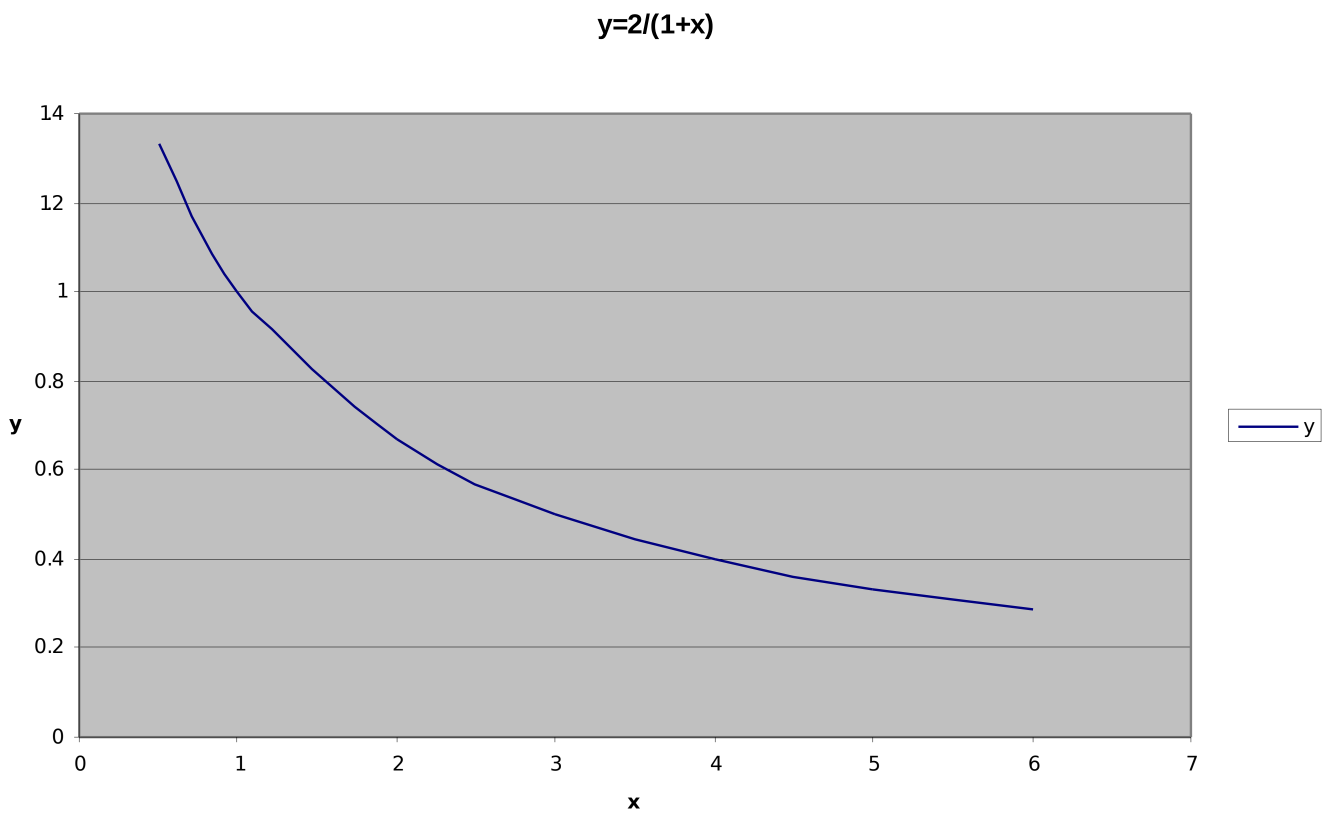
$$y = f(x) = 2/(1+x) \quad x \geq 0$$

$$1 + x = 2/y \; or \; x = \left(2/y\right) - 1$$

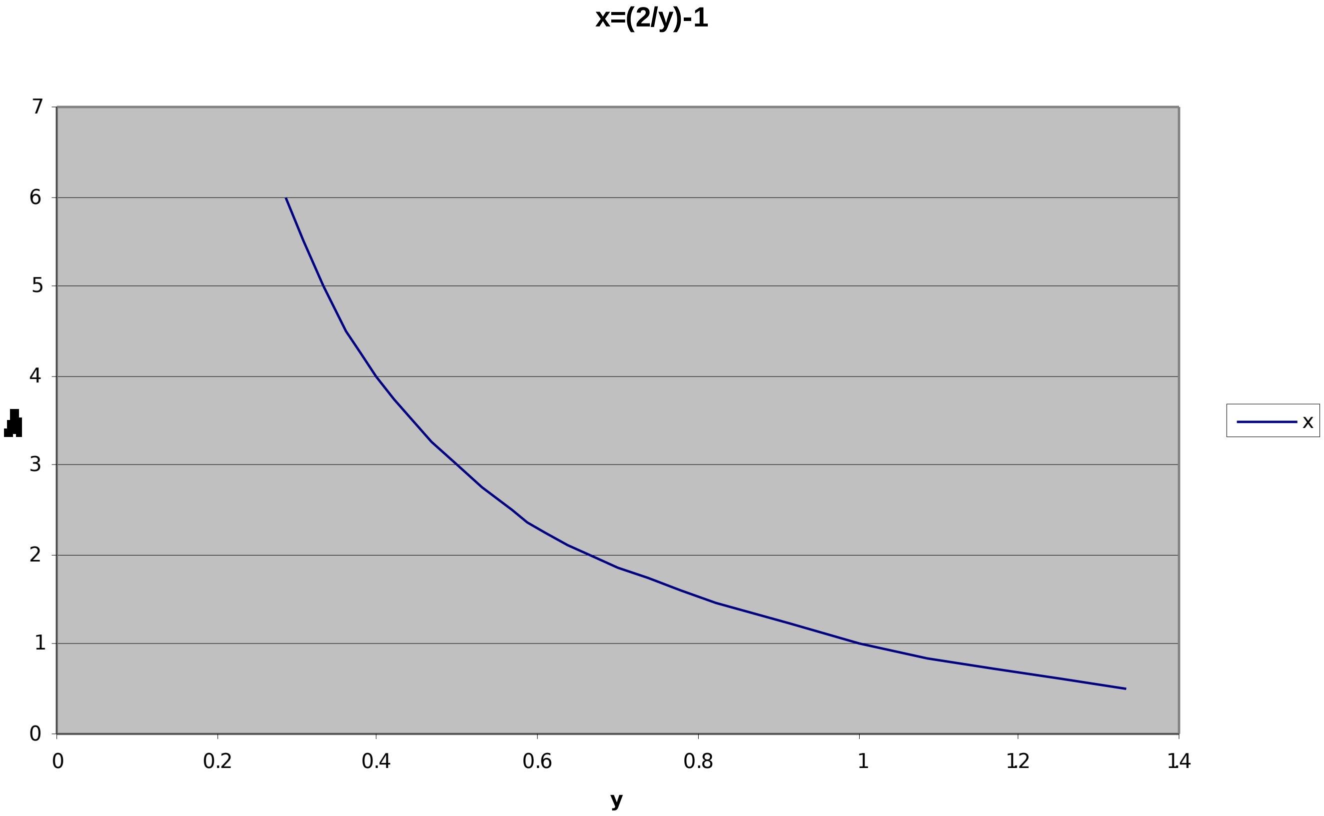
hence,

$$f^{-1}(x) = \left(2/x\right) - 1$$

Graph of example



Graph of inverse function



Special Property

An important property of a function *f(x)* and its inverse $f^{-1}(x)$ is

$$f\left(f^{-1}(x)\right) = x \quad and \quad f^{-1}\left(f(x)\right) = x.$$

For our example

$$f^{-1}\left(f(x)\right) = \left(2\big/\left(2\big/1+x\right) \right) - 1 = 1 + x - 1 = x$$

and

$$f\left(f^{-1}(x)\right) = 2\big/\left(1+\left(2\big/x - 1\right)\right) = 2\big/\left(2\big/x\right) = x$$

Composite Function

The term $f(f^{-1}(x))$ represents a composite function.

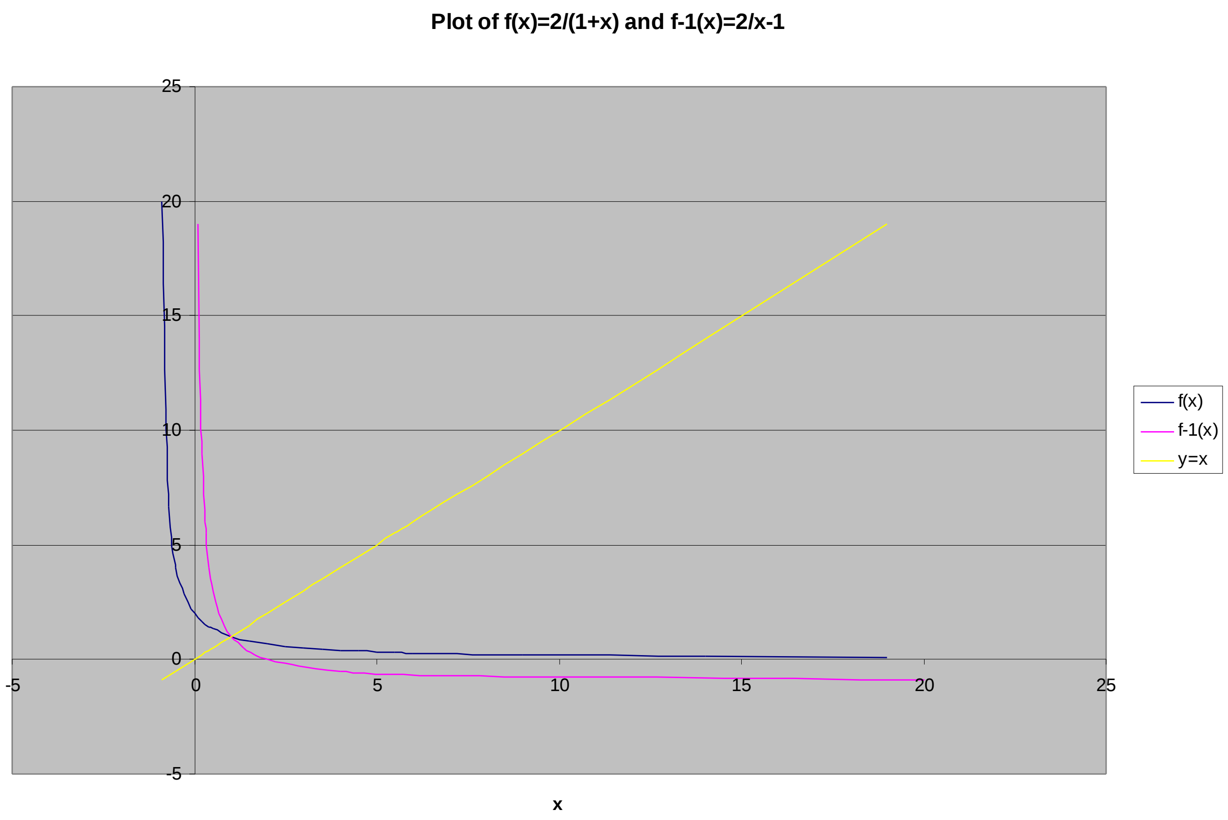
The argument of a composite function is itself a function.

$y = g(h(x))$ is a composite function with the inside function $h(x)$ and the outside function $g(\bullet)$ where the symbol $\bullet$ is a placeholder for the argument of the function g.

Graphing Inverse function

To graph the inverse function $y=f^{-1}(x)$ given the function $y=f(x)$, we make use of the fact that for any given ordered pair (a,b) associated with a one-to-one function there is an ordered pair (b,a) associated with the inverse function. To graph $y=f^{-1}(x)$, we make use of the 45° line, which is the graph of the function $y=x$ and passes through all points of the form (a,a). The graph of the function $y=f^{-1}(x)$, is the reflection of the graph of $f(x)$ across the line $y=x$.

Graph of function and inverse function



Common Example Inverse function

- Traditional demand function is written $q=f(p)$, i.e. quantity is function of price.
- Traditionally we graph the inverse function, $p=f^{-1}(q)$.
- Our Traditional Demand Curve shows maximum price people with pay to receive a given quantity.

Demand Function

A traditional demand function is

$$q = 10P^{-2}$$

This is a traditional constant elasticity demand function with elasticity = - 2 everywhere.

The inverse function we plot is

$$\mathrm{p} = \sqrt{10}q^{-1/2}$$

Plot of our Demand function