# Economics 2301

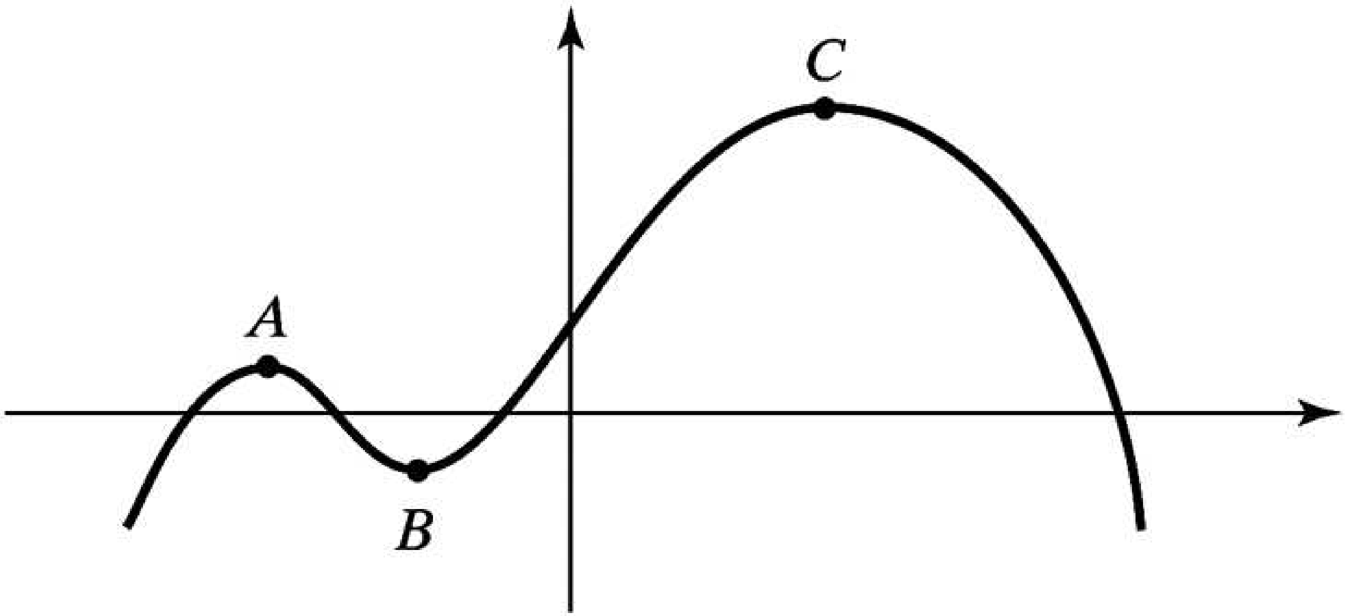
Lecture 4

Introduction to Functions Cont.

# Extreme Values

- The largest value of a function over its entire range is call its **Global maximum.**
- The smallest value over the entire range is called the **Global minimum.**
- Largest value within a small interval is a **local maximum.**
- Smallest value within a small interval is called a **local minimum.**

# **Figure 2.8** A Function with Extreme Values



# Average Rate of Change

The average rate of change of a function over some interval is the ratio of the change of the value of the dependent variable to the change in the value of the independent variable over that interval.

The average rate of change of the function $y = f(x)$ over the closed interval $[x_A, x_B]$ is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_B) - f(x_A)}{x_B - x_A}$$

# Average Rate of Change Linear Function

$Let\ y = \alpha + \beta x$

for the interval $[x_A, x_B]$, the average rate of change is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_B) - f(x_A)}{x_B - x_A} = \frac{(\alpha + \beta x_B) - (\alpha + \beta x_A)}{x_B - x_A}$$

$$= \frac{\beta(x_B - x_A)}{x_B - x_A} = \beta$$

This is the slope of our linear function.

# Average Rate of Change for Nonlinear function

Consider our function from lecture 6

$$y = \frac{2}{1+x}$$

Interval $[1,2]$

$$\frac{\Delta y}{\Delta x} = \frac{(2/1+2)-(2/1+1)}{2-1} = \frac{2/3-1}{1} = \frac{-1/3}{1} = -1/3$$

Interval $[1,5]$

$$\frac{\Delta y}{\Delta x} = \frac{(2/1+5)-(2/1+1)}{5-1} = \frac{2/6-1}{4} = \frac{-2/3}{4} = -1/6$$

For nonlinear function, average rate of change not the same over all intervals.

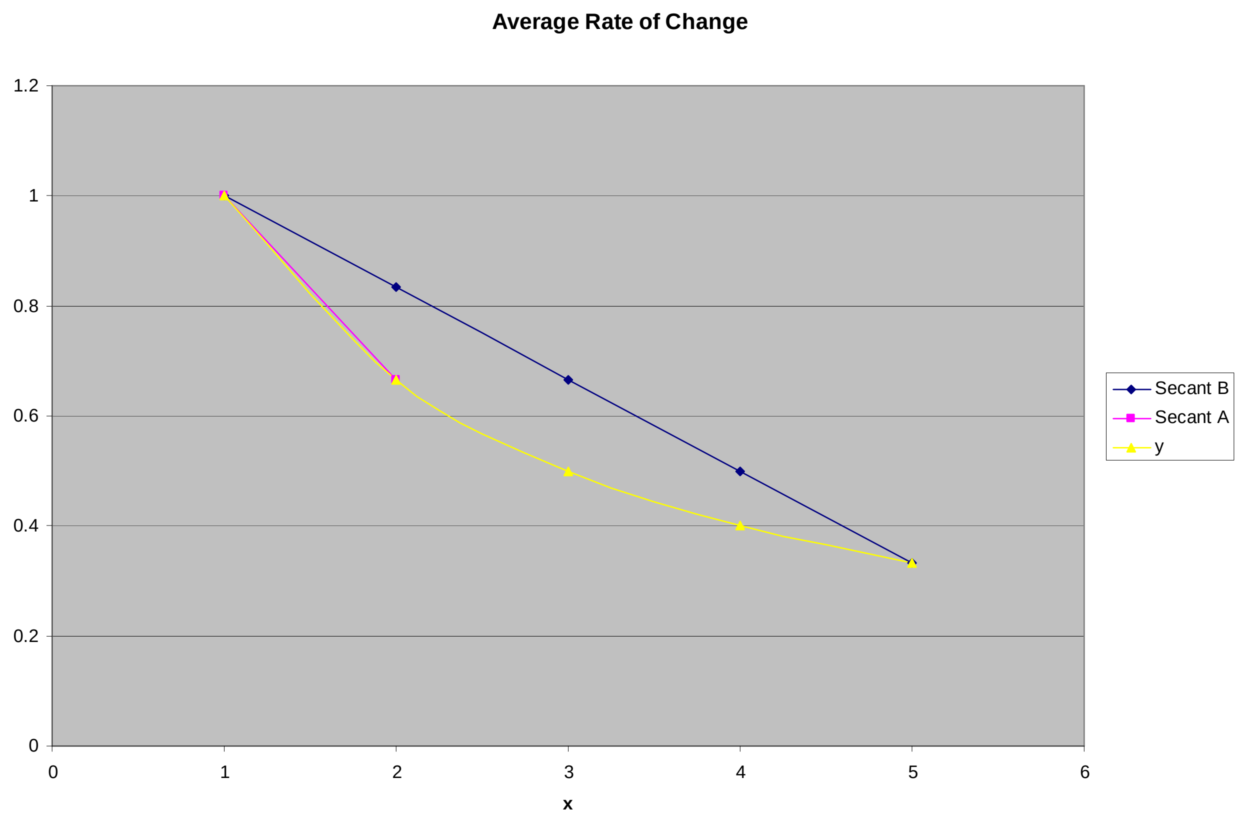
# Secant Line

Average rate of change of a function over some interval can be deplicted using a secant line. A secant line connects two points on the graph of a function with a straight line. Any point $(x',y')$ on the secant line connecting the two points $(x_A, y_A)$ and $(x_B, y_B)$ will satisfy the equation

$$(y'-y_A) = \left[\frac{f(x_B) - f(x_A)}{x_B - x_A}\right](x' - x_A) \; or$$

$$y' = y_A + \left[\frac{f(x_B) - f(x_A)}{x_B - x_A}\right](x' - x_A)$$

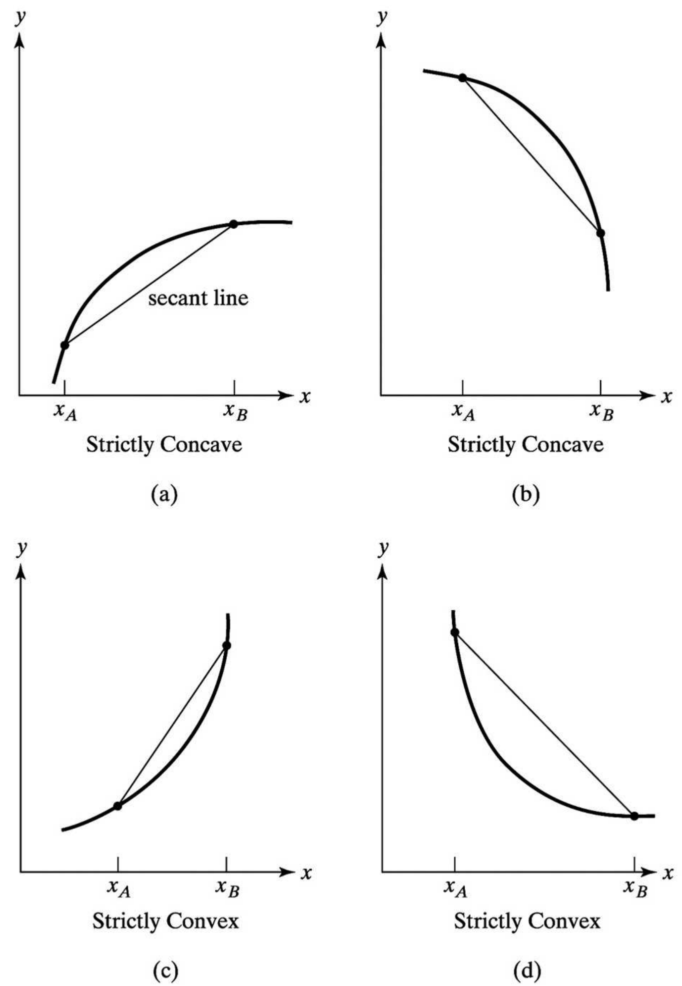
# Graph of Average Rate of Change for Nonlinear Function



# Concavity and Convexity

- The **concavity** of a univariate function is reflect by the shape of it graph and its secant line.
- Function is **strictly concave** in the interval $[x_A, x_B]$ if the secant line drawn on that interval lies wholly below the function.
- Function is **strictly convex** in the interval $[x_A, x_B]$ if the secant line drawn on that interval lies wholly above the function.

# Figure 2.10 Strictly Concave and Strictly Convex Functions



# Strictly Concave

The function $f(x)$ is strictly concave in an interval if, for any two distinct points $x^A$ and $x^B$ in that interval, and for all values of $\lambda$ in the open interval $(0,1)$,

$$f\left(\lambda x^A + (1-\lambda)x^B\right) > \lambda f\left(x^A\right) + (1-\lambda) f\left(x^B\right)$$

# Strictly Convex

The function $f(x)$ is strictly concex in an interval if, for any two distinct points $x^A$ and $x^B$ in that interval, and for all values of $\lambda$ in the open interval $(0,1)$,

$$f\left(\lambda x^A + (1-\lambda)x^B\right) < \lambda f\left(x^A\right) + (1-\lambda) f\left(x^B\right)$$

# Convexity and secant line

for a given value of $\lambda$ in the interval $(0,1)$, the argrument of the function in the interval $(x^A, x^B)$ is $x' = \lambda x^A + (1\text{-}\lambda)x^B$, where $x^A < x' < x^B$. The value of the function at the point $x'$ is $f(x') = f\left(\lambda x^A + (1\text{-}\lambda)x^B\right)$. With the definition of secant line the value on the secant line at $x'$ is $y'$

$$(y' - y^A) = \left[\frac{f(x^B) - f(x^A)}{x^B - x^A}\right]\left(\left[\lambda x^A + (1\text{-}\lambda)x^B\right] - x^A\right)$$

$$= \left[\frac{f(x^B) - f(x^A)}{x^B - x^A}\right](1-\lambda)\left(x^B - x^A\right) = (1-\lambda)\left(f(x^B) - f(x^A)\right)$$

since $y^A = f(x^A)$, if we add $f(x^A)$ to each side, we get

$$y' = \lambda f(x^A) + (1-\lambda) f(x^B)$$

# Our Example

Consider our function $y = \dfrac{2}{1+x}$ over the interval $(1{,}5)$.

Let $\lambda = 1/2$, so $x' = 3$.

$$\lambda f\left(x_A\right) + (1-\lambda)\, f\left(x_A\right) = \frac{1}{2}(1) + \frac{1}{2}\left(\frac{1}{3}\right) = \frac{2}{3}$$

$$f\left(\lambda x_A + (1-\lambda)x_B\right) = f\left(3\right) = \frac{1}{2} < \frac{2}{3}$$

This is true for all values of $\lambda$ in the interval $(0,1)$, hence our function is strictly convex over the interval $(1,5)$.

# Concave

The function $f(x)$ is concave in an interval if, for any two distinct points $x^A$ and $x^B$ in that interval, and for all values of $\lambda$ in the open interval $(0,1)$,

$$f\left(\lambda x^A + (1-\lambda)x^B\right) \geq \lambda f\left(x^A\right) + (1-\lambda) f\left(x^B\right)$$

# Convex

The function *f(x)* is concex in an interval if, for any two distinct points $x^A$ and $x^B$ in that interval, and for all values of $\lambda$ in the open interval $(0,1)$,

$$f\left(\lambda x^A + (1-\lambda)x^B\right) \leq \lambda f\left(x^A\right) + (1-\lambda) f\left(x^B\right)$$

# Figure 2.11 Concave and Convex Functions