Lecture 10

Solving Simultaneous Equations
Comparative Statics

- Economic analysis often considers the effect of a change in an exogenous variable on one or more endogenous variables.

- **Comparative Statics** compares the value of endogenous variables in one equilibrium to their value in another equilibrium.

- “**Statics**” refers to the fact that the analysis is not concerned with the way in which the new equilibrium is reached.
Comparative Statics

- Usually we vary one exogenous variable at a time in order to isolate the effect of that particular variable.
- The effort looks at the effect of a change in a variable, *ceteris paribus*, that is other things being equal.
Ask ourselves the question of how a change in the nominal money supply effects equilibrium income.

\[ \Delta \ln(M) = \ln(M_1) - \ln(M_0) \]  
For the ceteris paribus condition, assume \( \Delta \ln(G) = \ln(G_1) - \ln(G_0) = 0, \Delta \ln(P) = \ln(P_1) - \ln(P_0) = 0 \)

\[ \Delta \ln(Y) = \ln(Y_1) - \ln(Y_0) \]

\[
\begin{align*}
\frac{\alpha - \lambda}{\phi + \beta} + \frac{\gamma}{\phi + \beta} \ln(G_1) + \frac{\theta}{\phi + \beta} \ln(M_1) - \frac{\theta}{\phi + \beta} \ln(P_1) \\
- \left[ \frac{\alpha - \lambda}{\phi + \beta} + \frac{\gamma}{\phi + \beta} \ln(G_0) + \frac{\theta}{\phi + \beta} \ln(M_0) - \frac{\theta}{\phi + \beta} \ln(P_0) \right]
\end{align*}
\]

\[
\begin{align*}
= \frac{\gamma}{\phi + \beta} \Delta \ln(G) + \frac{\theta}{\phi + \beta} \Delta \ln(M) - \frac{\theta}{\phi + \beta} \Delta \ln(P) \\
= \frac{\gamma}{\phi + \beta} (0) + \frac{\theta}{\phi + \beta} \Delta \ln(M) - \frac{\theta}{\phi + \beta} (0) = \frac{\theta}{\phi + \beta} \Delta \ln(M)
\end{align*}
\]
Interest Rate Comparative Statics

Simple way to do comparative statics is to write the difference equation for the reduced form equation. For interest rate in the IS-LM model, we get

\[ \Delta \ln(i) = \frac{\phi \gamma}{\phi + \beta} \Delta \ln(G) - \frac{\beta \phi}{\phi + \beta} \Delta \ln(M) + \frac{\beta \phi}{\phi + \beta} \Delta \ln(P) \]

Now set all the changes in the exogenous variables equal to zero except the change of interest (i.e. change in money) then our comparative static result is

\[ \Delta \ln(i) = -\frac{\beta \phi}{\phi + \beta} \Delta \ln(M) \]
For the demand and supply model, the difference equations for the equilibrium price and quantity are

\[ \Delta P = \frac{\gamma}{\phi + \beta} \Delta I + \frac{\theta}{\phi + \beta} \Delta P^s + \frac{\phi}{\phi + \beta} \Delta W \]

\[ \Delta Q = \frac{\phi \gamma}{\phi + \beta} \Delta I + \frac{\phi \theta}{\phi + \beta} \Delta P^s - \frac{\phi \beta}{\phi + \beta} \Delta W \]
Incidence of a Tax

We are going to look at the distribution of the burden of a excise tax on a good.

\[ P_C = P_p + T \]
\[ Q^D = \alpha - \beta P_C + \gamma G \]
\[ Q^S = \theta + \lambda P_p - \phi N \]

Equilibrium: \( Q^D = Q^S = Q \)
\[ \theta + \lambda P_p - \phi N = \alpha - \beta (P_p + T) + \gamma G \]
\[ (\lambda + \beta)P_p = (\alpha - \theta) - \beta T + \gamma G + \phi N \]

\[ P_p = \frac{\alpha - \theta}{\lambda + \beta} - \frac{\beta}{\lambda + \beta} T + \frac{\gamma}{\lambda + \beta} G + \frac{\phi}{\lambda + \beta} N \]
Incidence of a Tax Continued

\[ P_C = P_P + T = \frac{\alpha - \theta}{\lambda + \beta} - \frac{\beta}{\lambda + \beta} T + \frac{\gamma}{\lambda + \beta} G + \frac{\phi}{\lambda + \beta} N + T \]

\[ = \frac{\alpha - \theta}{\lambda + \beta} + \frac{\lambda}{\lambda + \beta} T + \frac{\gamma}{\lambda + \beta} G + \frac{\phi}{\lambda + \beta} N \]

Comparative Statics of a change in the Tax

\[ \Delta P_P = - \frac{\beta}{\lambda + \beta} \Delta T \]

\[ \Delta P_C = \frac{\lambda}{\lambda + \beta} \Delta T \]