



Economics 2301

Lecture 10

Solving Simultaneous Equations

[Comparative Statics]

- Economic analysis often considers the effect of a change in an exogenous variable on one or more endogenous variables.
- **Comparative Statics** compares the value of endogenous variables in one equilibrium to their value in another equilibrium.
- “**Statics**” refers to the fact that the analysis is not concerned with the way in which the new equilibrium is reached.

[Comparative Statics]

- Usually we vary one exogenous variable at a time in order to isolate the effect of that particular variable.
- The effort looks at the effect of a change in a variable, *ceteris paribus*, that is other things being equal.

Comparative Statics for IS-LM

Ask ourselves the question of how a change in the nominal money supply effects equilibrium income.

$\Delta \ln(M) = \ln(M_1) - \ln(M_0)$ For the ceteris paribus condition, assume $\Delta \ln(G) = \ln(G_1) - \ln(G_0) = 0$, $\Delta \ln(P) = \ln(P_1) - \ln(P_0) = 0$

$$\Delta \ln(Y) = \ln(Y_1) - \ln(Y_0)$$

$$\begin{aligned} &= \frac{\alpha - \lambda}{\phi + \beta} + \frac{\gamma}{\phi + \beta} \ln(G_1) + \frac{\theta}{\phi + \beta} \ln(M_1) - \frac{\theta}{\phi + \beta} \ln(P_1) \\ &- \left[\frac{\alpha - \lambda}{\phi + \beta} + \frac{\gamma}{\phi + \beta} \ln(G_0) + \frac{\theta}{\phi + \beta} \ln(M_0) - \frac{\theta}{\phi + \beta} \ln(P_0) \right] \\ &= \frac{\gamma}{\phi + \beta} \Delta \ln(G) + \frac{\theta}{\phi + \beta} \Delta \ln(M) - \frac{\theta}{\phi + \beta} \Delta \ln(P) \\ &= \frac{\gamma}{\phi + \beta} (0) + \frac{\theta}{\phi + \beta} \Delta \ln(M) - \frac{\theta}{\phi + \beta} (0) = \frac{\theta}{\phi + \beta} \Delta \ln(M) \end{aligned}$$

Interest Rate Comparative Statics

Simple way to do comparative statics is to write the difference equation for the reduced form equation. For interest rate in the IS - LM model, we get

$$\Delta \ln(i) = \frac{\phi}{\phi + \beta} \gamma \Delta \ln(G) - \frac{\beta \phi}{\phi + \beta} \Delta \ln(M) + \frac{\beta \phi}{\phi + \beta} \Delta \ln(P)$$

Now set all the changes in the exogenous variables equal to zero except the change of interest (i.e. change in money) then our comparative static result is

$$\Delta \ln(i) = -\frac{\beta \phi}{\phi + \beta} \Delta \ln(M)$$

Difference equations: Demand and Supply

For the demand and supply model, the difference equations for the equilibrium price and quantity are

$$\Delta P = \frac{\gamma}{\phi + \beta} \Delta I + \frac{\theta}{\phi + \beta} \Delta P^s + \frac{\varphi}{\phi + \beta} \Delta W$$

$$\Delta Q = \frac{\phi \gamma}{\phi + \beta} \Delta I + \frac{\phi \theta}{\phi + \beta} \Delta P^s - \frac{\phi \beta}{\phi + \beta} \Delta W$$

Incidence of a Tax

We are going to look at the distribution of the burden of an excise tax on a good.

$$P_C = P_P + T$$

$$Q^D = \alpha - \beta P_C + \gamma G$$

$$Q^S = \theta + \lambda P_P - \phi N$$

$$\text{Equilibrium: } Q^D = Q^S = Q$$

$$\theta + \lambda P_P - \phi N = \alpha - \beta(P_P + T) + \gamma G$$

$$(\lambda + \beta)P_P = (\alpha - \theta) - \beta T + \gamma G + \phi N$$

$$P_P = \frac{\alpha - \theta}{\lambda + \beta} - \frac{\beta}{\lambda + \beta} T + \frac{\gamma}{\lambda + \beta} G + \frac{\phi}{\lambda + \beta} N$$

Incidence of a Tax Continued

$$\begin{aligned} P_C &= P_P + T = \frac{\alpha - \theta}{\lambda + \beta} - \frac{\beta}{\lambda + \beta} T + \frac{\gamma}{\lambda + \beta} G + \frac{\phi}{\lambda + \beta} N + T \\ &= \frac{\alpha - \theta}{\lambda + \beta} + \frac{\lambda}{\lambda + \beta} T + \frac{\gamma}{\lambda + \beta} G + \frac{\phi}{\lambda + \beta} N \end{aligned}$$

Comparative Statics of a change in the Tax

$$\Delta P_P = -\frac{\beta}{\lambda + \beta} \Delta T$$

$$\Delta P_C = \frac{\lambda}{\lambda + \beta} \Delta T$$