Economics 2301

Lecture 14

Differential Calculus

Need for Differential Calculus

- We have seen the contribution of Comparative statics to our understanding of economics.
- We have learned to do comparative statics for linear models.
- Differential calculus will permit us to do comparative statics for nonlinear models.

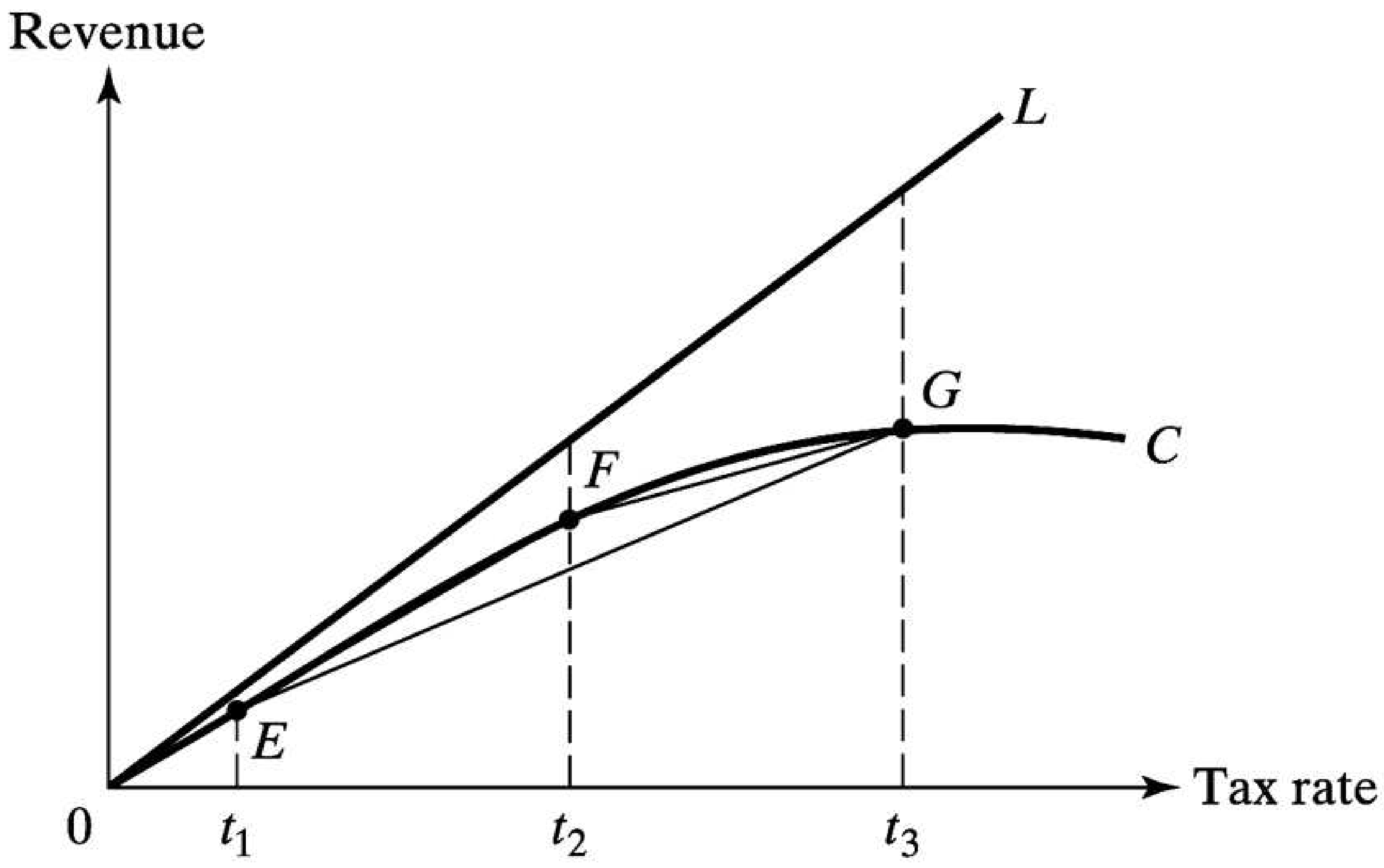
Functions and Rates of Change

Assume we have an excise tax on a good, Q. The tax per unit sold is "t". Our revenue from the tax is R=tQ. We ask ourselves the question "How much revenue will we gain if we raise the tax from t_1 to t_2?" If there is no change in sales, revenue will be t_2Q. This makes revenue a linear function of the tax rate (see line OL on figure 6.1).

Increasing the excise tax, effectively increases the price of the good being sold. Law of Demand suggests that when price increases, the quantity sold will decrease. If this is true then the revenue from the excise tax after the rate increase will be less t_2Q. In other words, $\Delta R \neq Q\Delta t$ or the average rate of change $\Delta R/ \Delta t \neq Q$.

Our tax revenue function in general will increase at a decreasing rate and may max out at some tax rate. If this is true, our average rate or change or secant function will be different depending on the initial and final tax rate.

Figure 6.1 Tax Revenues and Tax Rates



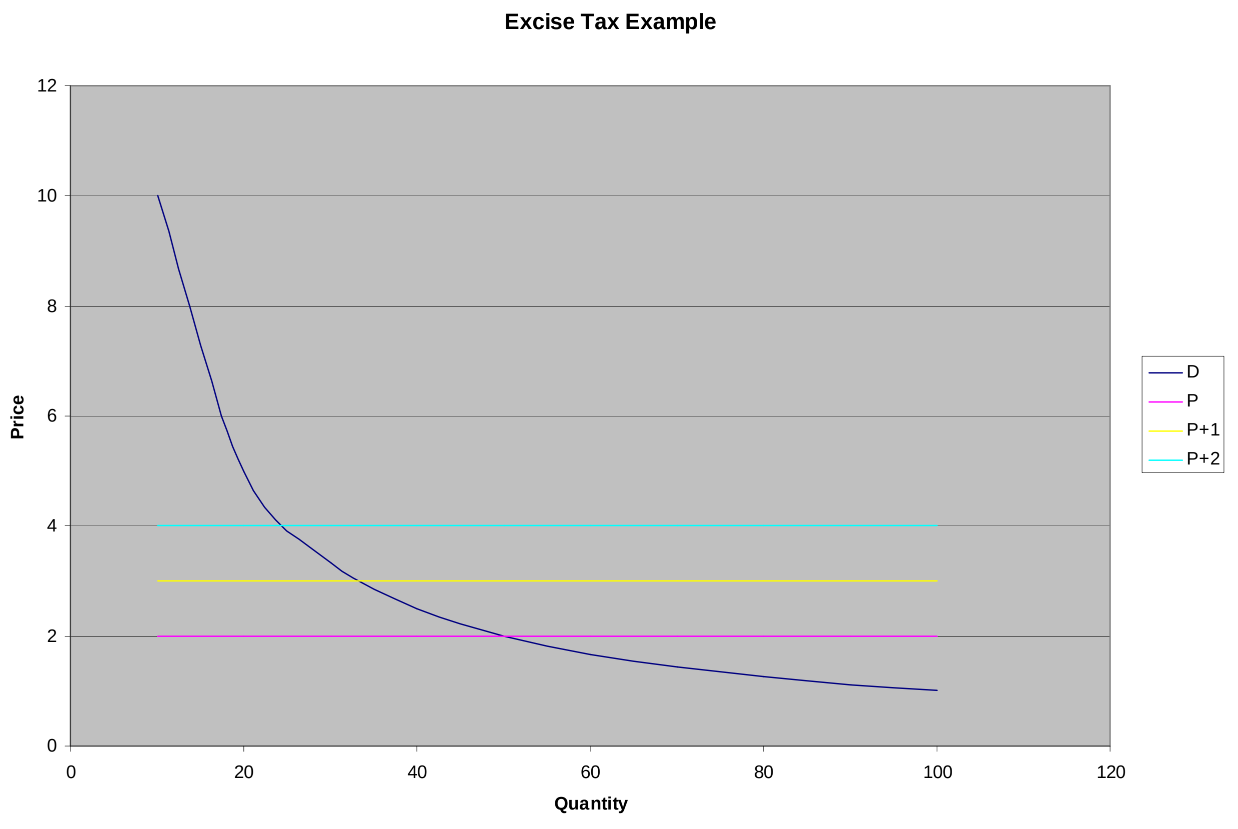
Example of constant elasticity demand function

Consider imposing excise tax on a good with a constant elasticity demand function. Further assume the supply of the good is perfectly elastic at the price P_P. With these assumptions, we can write our demand function as

$Q = \dfrac{C}{P_p + t}$ and our tax revenue function as

$R = tQ = \dfrac{tC}{P_p + t}$. This function increases at a decreasing rate in t.

Graph of Demand and tax



Graph of tax revenue function for constant elasticity demand