# Economics 2301

Lecture 21

Univariate Calculus

# M2 Multiplier

Let

Currency in the hands of the public (C)

Checkable deposits (D)

Small savings and time deposits (D*)

Required Reserves (RR)

Excess Reserves (ER)

$$\left| \begin{array}{c} \text{Overnight repurchase agreements} \\ \text{Overnight Eurodollars} \\ \text{Money Market Deposit accounts} \\ \text{Individual Money Market mutual funds} \end{array} \right\rangle = MI$$

$$M2 = C + D + D^{*} + MI$$

# M2 Multiplier

Taking the ratio of M2 to MB (Monetary Base) and multiplying top and bottom by 1/D, we get

$$\mathrm{M2/MB} = (\mathrm{C} + \mathrm{D} + \mathrm{D}^* + \mathrm{MI})/(\mathrm{RR} + \mathrm{ER} + \mathrm{C})$$

$$\mathrm{M2/MB} = ((1/\mathrm{D})M2)/((1/D)MB)$$

$$= (C/D + D/D + D^*/D + MI/D)/(RR/D + ER/D + C/D)$$

$$= (c + 1 + d + mi)/(r + e + c) = m2$$

*Where*

c = currency ratio, d = time deposit ratio

mi = monetary instruments ratio, r = required reserves ratio

e = excess reserve ratio. *m2* = M2 multiplier

# M2 Multiplier

The M2 supply equation becomes

$$M2 = m2MB$$

Using our concept of a differential

$$dM2 = m2 \cdot dMB$$

Question: How is the M2 differential influenced when one of the determinates of *m2* changes? In particular when the currency ratio changes. We want *dM2/dm2*

$$m2 = (1 + c + d + mi)/(r + e + c)$$

$$\frac{dm2}{dc} = \frac{(r+e+c)\cdot 1 - (1+c+d+mi)\cdot 1}{(r+e+c)^2} = \frac{(r+e) - (1+d+mi)}{(r+e+c)^2}$$

$$< 0$$

# Natural Logarithmic Function Rule

The derivative of $f(x) = \ln(x)$ is

$$f'(x) = \frac{d \ln x}{dx} = \frac{1}{x}$$

Proof :

Let $h(x) = e^{\ln(x)}$ $\quad$ *Note* $h(x) = x$ *and* $h'(x) = 1$

Now by the chain rule

$$h'(x) = \frac{d \ln(x)}{dx} \cdot e^{\ln(x)} = \frac{d \ln(x)}{dx} \cdot x = 1$$

$$\Rightarrow \frac{d \ln(x)}{dx} = \frac{1}{x}$$

*QED*

# Logarithmic Function Rule

The derivative of $f(x) = \log_b(x)$ is

$$\frac{d\log_b(x)}{dx} = \frac{1}{x} \cdot \frac{1}{\ln(b)}$$

Recall

$$\ln(x) = \ln(b)\log_b(x)$$

$$\Rightarrow \frac{d\ln(x)}{dx} = \ln(b)\frac{d\log_b(x)}{dx}$$

$$\Rightarrow \frac{d\log_b(x)}{dx} = \frac{d\ln(x)}{dx} \cdot \frac{1}{\ln(b)} = \frac{1}{x} \cdot \frac{1}{\ln(b)}$$

# Added Results

$$\text{recall} \log_{\mathrm{w}} H = \log_w J \cdot \log_J H$$

$$\Rightarrow 1 = \ln(e) = \ln(b) \cdot \log_b(e)$$

$$\Rightarrow \frac{1}{\ln(b)} = \log_b(e)$$

$$\Rightarrow \frac{d \log_b(x)}{dx} = \frac{1}{\ln(b)} \cdot \frac{1}{x} = \log_b(e) \cdot \frac{1}{x}$$

# General Natural Logarithmic Function Rule

The derivative of $f(x) = \ln(h(x))$ is

$$f'(x) = \frac{d \ln(h(x))}{dx} = \frac{h'(x)}{h(x)}$$

*Example*

$$let \;\; f(x) = \ln(x^2 + 2x) \quad then$$

$$f'(x) = \frac{2x+2}{x^2+2x}$$

# Another application of General natural logarithmic function rule

$$f(x) = \ln\left(\frac{g(x)}{h(x)}\right)$$

$$let\ u(x) = \frac{g(x)}{h(x)},\ then\ \ f(x) = \ln(u(x))$$

$$f'(x) = \frac{u'(x)}{u(x)} \qquad u'(x) = \frac{h(x)g'(x) - h'(x)g(x)}{\left[h(x)\right]^2}$$

$$\Rightarrow\ f'(x) = \frac{h(x)g'(x) - h'(x)g(x)}{\left[h(x)\right]^2} \Bigg/ \frac{g(x)}{h(x)}$$

$$= \frac{g'(x)}{g(x)} - \frac{h'(x)}{h(x)}$$

# Easier Derivation

$$Note\ \ f(x) = \ln\left(\frac{g(x)}{h(x)}\right) = \ln(g(x)) - \ln(h(x))$$

$$f'(x) = \frac{d\ln(g(x))}{dx} - \frac{d\ln(h(x))}{dx} = \frac{g'(x)}{g(x)} - \frac{h'(x)}{h(x)}$$

$$Example: Let\ \ f(x) = \ln\left(\frac{e^{rx}}{x^r}\right)$$

$$f'(x) = \frac{re^{rx}}{e^{rx}} - \frac{rx^{r-1}}{x^r} = r - \frac{r}{x} = \frac{r(x-1)}{x}$$

Show me an easier way to get this result.

# Purchasing Power Parity

Under the theory of purchasing power parity, the equilibrium exhange rate for the dollar at a point t in time is $E(t) = \frac{P_f(t)}{P_{US}(t)}$

where $P_f(t) =$ Foreign price level at time t, and $P_{US}(t) =$ U.S. price level at point t. Question : What condition must hold for the exchange rate to remain constant over time?

$$\frac{\mathrm{d}E(t)}{dt} = 0. \quad Note\ \ln\left(E(t)\right) = \ln\left(P_f(t)\right) - \ln\left(P_{US}(t)\right)$$

$$\frac{\mathrm{d}E(t)}{dt} = 0 \Rightarrow \frac{d\ln\left(E(t)\right)}{dt} = \frac{1}{E(t)} \cdot \frac{\mathrm{d}E(t)}{dt} = 0$$

$\frac{1}{E(t)} \cdot \frac{\mathrm{d}E(t)}{dt} =$ instantaneous rate of change of the exhange rate.

# Purchasing Power Parity Cont.

$$\text{Recall } \ln\left(E(t)\right) = \ln\left(P_f(t)\right) - \ln\left(P_{US}(t)\right), \textit{then}$$

$$\frac{d\ln\left(E(t)\right)}{dt} = \frac{dE(t)}{dt} \Big/ E(t) = \frac{dP_f(t)}{dt} \Big/ P_f(t) - \frac{dP_{US}(t)}{dt} \Big/ P_{US}(t)$$

$$\frac{dE(t)}{dt} \Big/ E(t) = 0 \Rightarrow \frac{dP_f(t)}{dt} \Big/ P_f(t) = \frac{dP_{US}(t)}{dt} \Big/ P_{US}(t)$$

$$\Rightarrow What\,?$$