Economics 2301

Lecture 22

Elasticity

Elasticity

- An **elasticity** measures a specific form of responsiveness.
- The percentage change in one variable that accompanies a 1% change in another variable.
- An elasticity is not affected by the units used to measure variables.

Elasticity

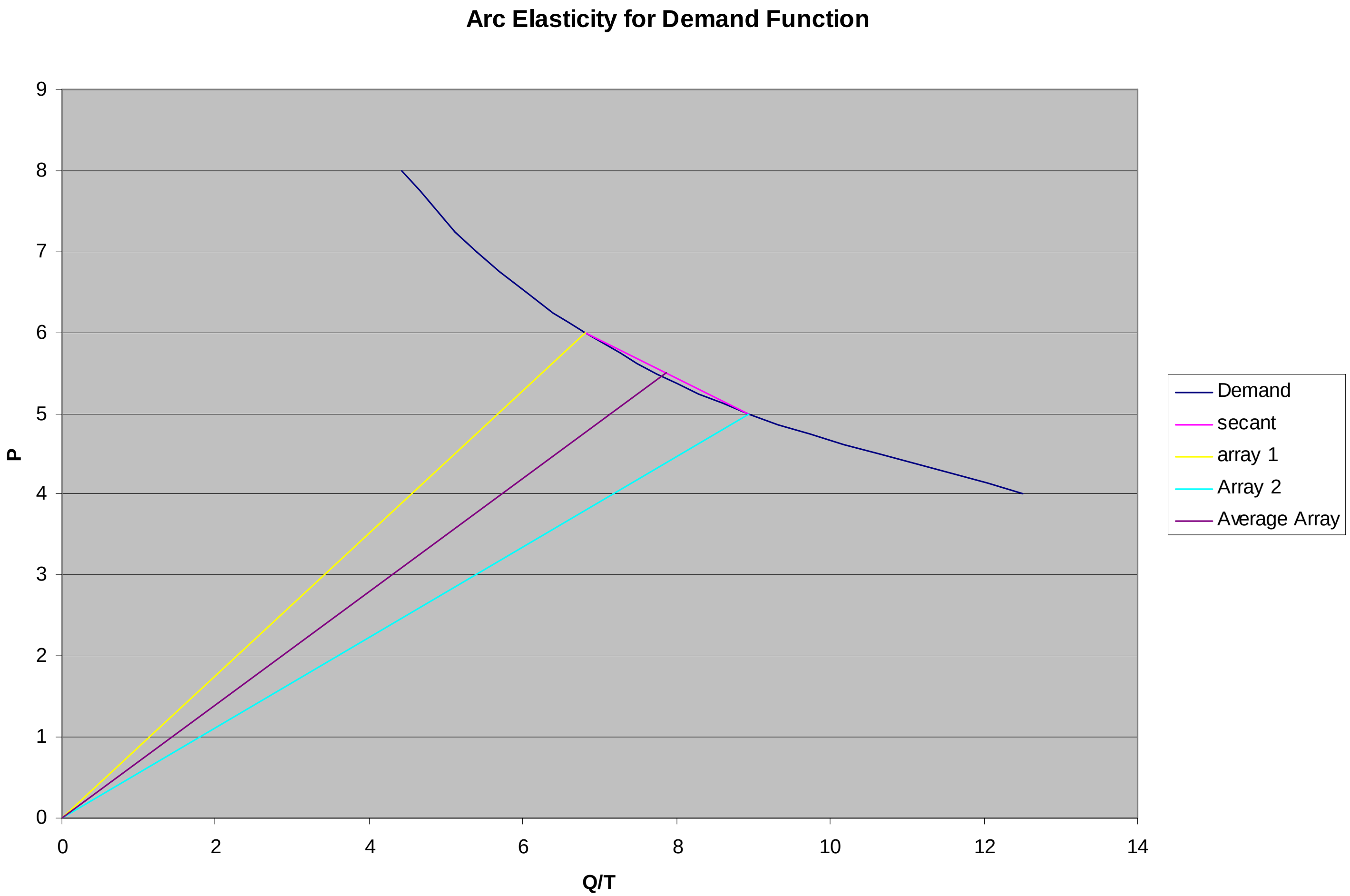
Arc elasticity between y and x is

$$\varepsilon_{y \cdot x} = \frac{\Delta y / y}{\Delta x / x} = \frac{\Delta y}{\Delta x} \cdot \frac{x}{y} = \frac{\Delta y}{\Delta x} \bigg/ \frac{y}{x} = difference\ quotient / Array$$

Point Elasticity

$$\varepsilon_{y \cdot x} = \frac{dy / y}{dx / x} = \frac{dy}{dx} \cdot \frac{x}{y} = \frac{dy}{dx} \bigg/ \frac{y}{x} = \text{marginal/average}$$

Arc Elasticity



Arc Elasticity

Price	Quantity
$8	4.419
$7	5.399
$6	6.804
$5	8.944
$4	12.50

Arc Elasticity from P=6 to P=5

Arc Elasticity for Starting Values

$$\varepsilon_{q\cdot p} = \frac{\Delta q}{\Delta p} \Big/ \frac{q_0}{p_0} = \frac{8.944 - 6.804}{5-6} \Big/ \frac{6.804}{6} = -2.14/1.134 = -1.887$$

Arc Elasticity for Ending Values

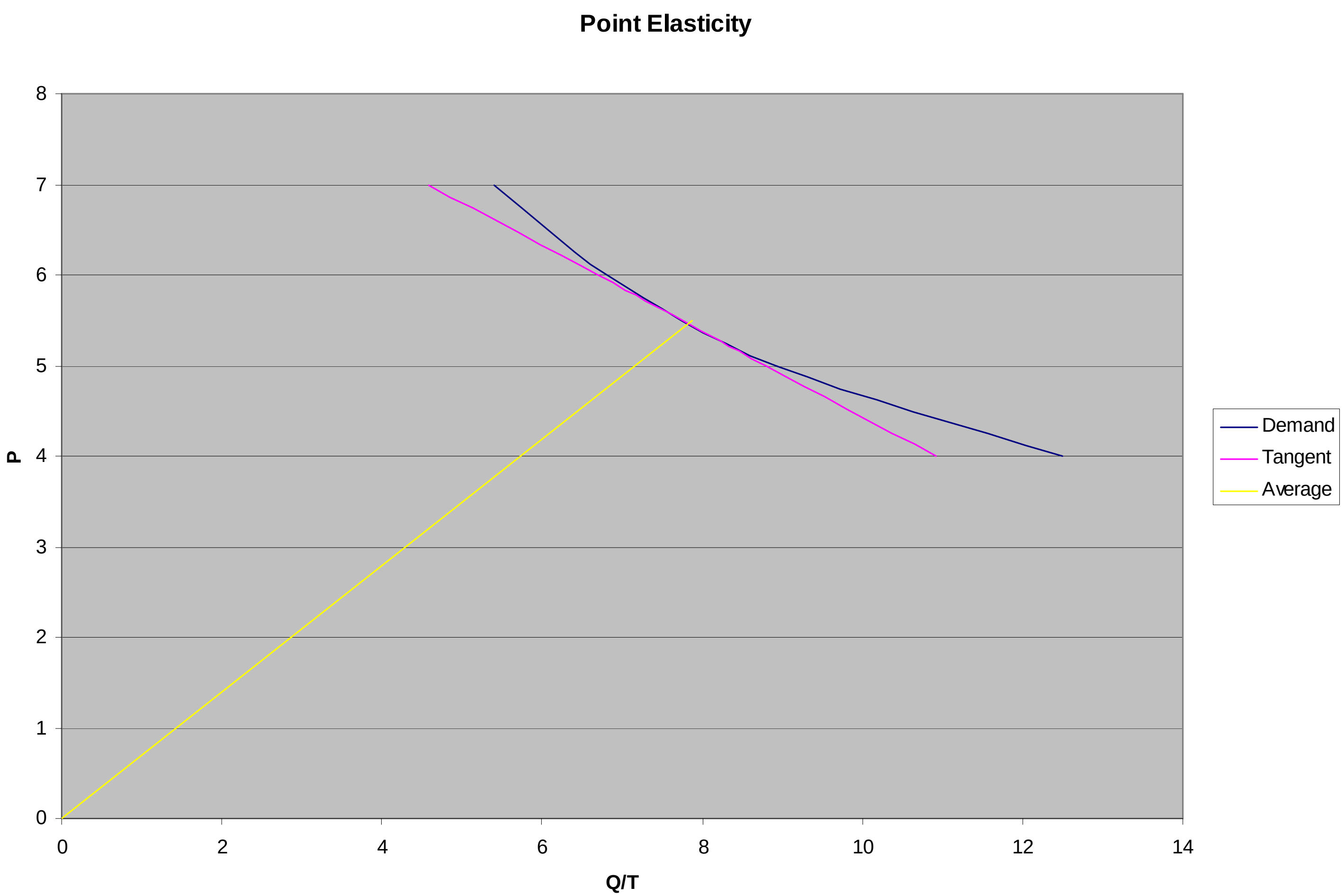
$$\varepsilon_{q\cdot p} = \frac{\Delta q}{\Delta p} \Big/ \frac{q_1}{p_1} = \frac{8.944 - 6.804}{5-6} \Big/ \frac{8.944}{5} = -2.14/1.7888 = -1.196$$

Arc Elasticity for Average Values

$$\varepsilon_{q\cdot p} = \frac{\Delta q}{\Delta p} \Big/ \frac{(q_0 + q_1)/2}{(p_0 + p_1)/2} = \frac{8.944 - 6.804}{5-6} \Big/ \frac{(6.804 + 8.944)/2}{(6+5)/2}$$

$$= -2.14/(7.874/5.5) = -2.14/1.4316 = -1.495$$

Point Elasticity at P=$5.50



Point Elasticity our Demand Function

Our demand function is

$$Q = 100P^{-1.5}$$

$$\varepsilon_{q \cdot p} = \frac{dQ}{dP} \Big/ \frac{Q}{P} = -1.5 \cdot 100P^{-2.5} \Big/ \frac{Q}{P} = -1.5 \cdot 100P^{-2.5} \Big/ \frac{100P^{-1.5}}{P}$$

$$= -1.5 \cdot 100P^{-2.5} / 100P^{-2.5} = -1.5$$

Point Elasticity for Constant Elasticity Demand Function

The constant elasticity demand function can be written

$$Q = \alpha P^{-\beta}$$

$$\varepsilon_{Q \cdot P} = {dQ}/{dP} \Big/ \frac{Q}{P} = -\beta \cdot \alpha P^{-\beta-1} \Big/ \frac{Q}{P} = -\beta \cdot \alpha P^{-\beta-1} \Big/ \frac{\alpha P^{-\beta}}{P}$$

$$= -\beta \cdot \alpha P^{-\beta-1} / \alpha P^{-\beta-1} = -\beta$$

Elasticities

- The terms elastic, unit elastic, and inelastic represent elasticities that are, in absolute value, greater than 1, equal to 1, and less than 1, respectively.

Linear Demand Function

$$let\ q = \alpha - \beta p$$

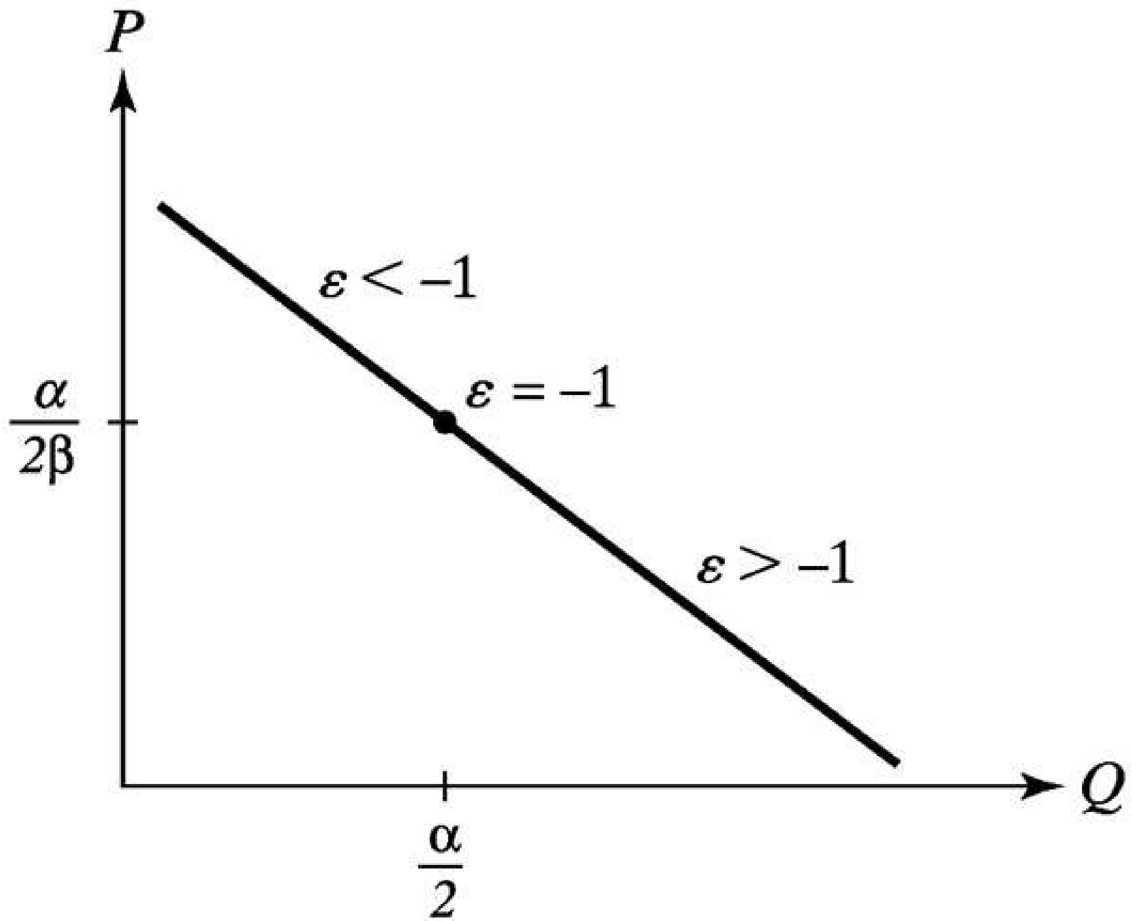
$$\varepsilon_{q \cdot p} = \frac{dq}{dp} \cdot \frac{p}{q} = -\beta \cdot \frac{p}{q} = -\beta / \frac{q}{p}$$

$$\varepsilon_{q \cdot p} = -1\,(unitary\ elasticity) \quad when$$

$$-\beta / \frac{q}{p} = -1 \Rightarrow \beta p = q \Rightarrow \beta p = \alpha - \beta p$$

$$\Rightarrow 2\beta p = \alpha \Rightarrow p = \frac{\alpha}{2\beta}\ and\ q = \alpha - \beta \frac{\alpha}{2\beta} = \frac{\alpha}{2}$$

Figure 7.1 Elasticities of a Linear Demand Function



General Elasticity Rule

$$let\ q = f(p) = f(e^{\ln p})$$

$$let\ u = \ln p \quad then\ q = f(e^u) \quad and$$

$$\ln q = \ln(f(e^u))$$

$$\frac{d \ln q}{du} = \frac{1}{f(e^u)} f'(e^u)e^u = \frac{1}{q} \cdot \frac{dq}{dp} \cdot p = \frac{p}{q} \cdot \frac{dq}{dp} = \varepsilon_{q \cdot p}$$

$$note: \quad \frac{d \ln q}{du} = \frac{d \ln q}{d \ln p}$$

$$\therefore \text{in general}\ \varepsilon_{q \cdot p} = \frac{d \ln q}{d \ln p}$$

Constant elasticity demand function

The constant elasticity demand function is

$$q = \alpha p^{-\beta} \Rightarrow \ln q = \ln \alpha - \beta \ln p$$

$$\varepsilon_{q \cdot p} = \frac{d \ln q}{d \ln p} = -\beta$$

The same result we obtained earlier.

Tax Incidence Example

$$\text{Let } Q^D = Q^D(P_c),\ \ Q^S = Q^S(P_P)\ and\ P_c = P_P + T$$

$$\text{Equilibrium}: Q^D(P_c) = Q^S(P_P)$$

Taking differential of both sides, we get

$$\frac{dQ^D}{dP_C} dP_C = \frac{dQ^S}{dP_P} dP_P\ and\ dP_c = dP_P + dT,\ thus$$

$$\frac{dQ^D}{dP_C}(dP_P + dT) = \frac{dQ^S}{dP_P} dP_P\ or\ \left(\frac{dQ^D}{dP_C} - \frac{dQ^S}{dP_P}\right) dP_P = -\frac{dQ^D}{dP_C} dT$$

At the initial tax rate of zero, we have $P_C = P_P = P$

$and\ Q^D = Q^S = Q.$

Tax Incidence continued

Multipling the expression from previous slide by *P/Q*, we get

$$\left(\frac{dQ^D}{dP_C}\cdot\frac{P}{Q}-\frac{dQ^S}{dP_P}\cdot\frac{P}{Q}\right)dP_P=-\frac{dQ^D}{dP_C}\cdot\frac{P}{Q}dT$$

$Let\ \varepsilon^D=\dfrac{dQ^D}{dP_C}\cdot\dfrac{P}{Q}\ and\ \varepsilon^S=\dfrac{dQ^S}{dP_P}\cdot\dfrac{P}{Q}$, then our expressing

becomes $\left(\varepsilon^D-\varepsilon^S\right)dP_P=-\varepsilon^D dT$

$$\Rightarrow\frac{dP_P}{dT}=-\frac{\varepsilon^D}{\left(\varepsilon^D-\varepsilon^S\right)}\Rightarrow -1<\frac{dP_P}{dT}<0 \text{ since } \varepsilon^D<0.$$

Tax Incidence continued

$$given\; dP_C = dP_P + dT \Rightarrow \frac{dP_C}{dT} = \frac{dP_P}{dT} + 1$$

$$\Rightarrow \frac{dP_C}{dT} = 1 - \frac{\varepsilon^D}{\varepsilon^D - \varepsilon^S} = -\frac{\varepsilon^S}{\varepsilon^D - \varepsilon^S}$$

$$\Rightarrow 0 < \frac{dP_C}{dT} < 1$$