Economics 2301

Lecture 23

Second Derivative

Need for Second Derivative

- Several concepts in economics concern changes in the marginal contribution of a variable.
  - Diminishing marginal Utility
  - Increasing Marginal Cost.
- These involves changes in the marginal function or the derivative of the original function.

Second Derivative

- The derivate of a function is itself a function.
- The derivative of the derivative of a function is the function's **second derivative.**

Second Derivative

The second derivative of a function $y = f(x)$, denoted as $f''(x)$ or d^2y/dx^2, is the derivative of its first derivative. That is,

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

Interpreting the second derivative

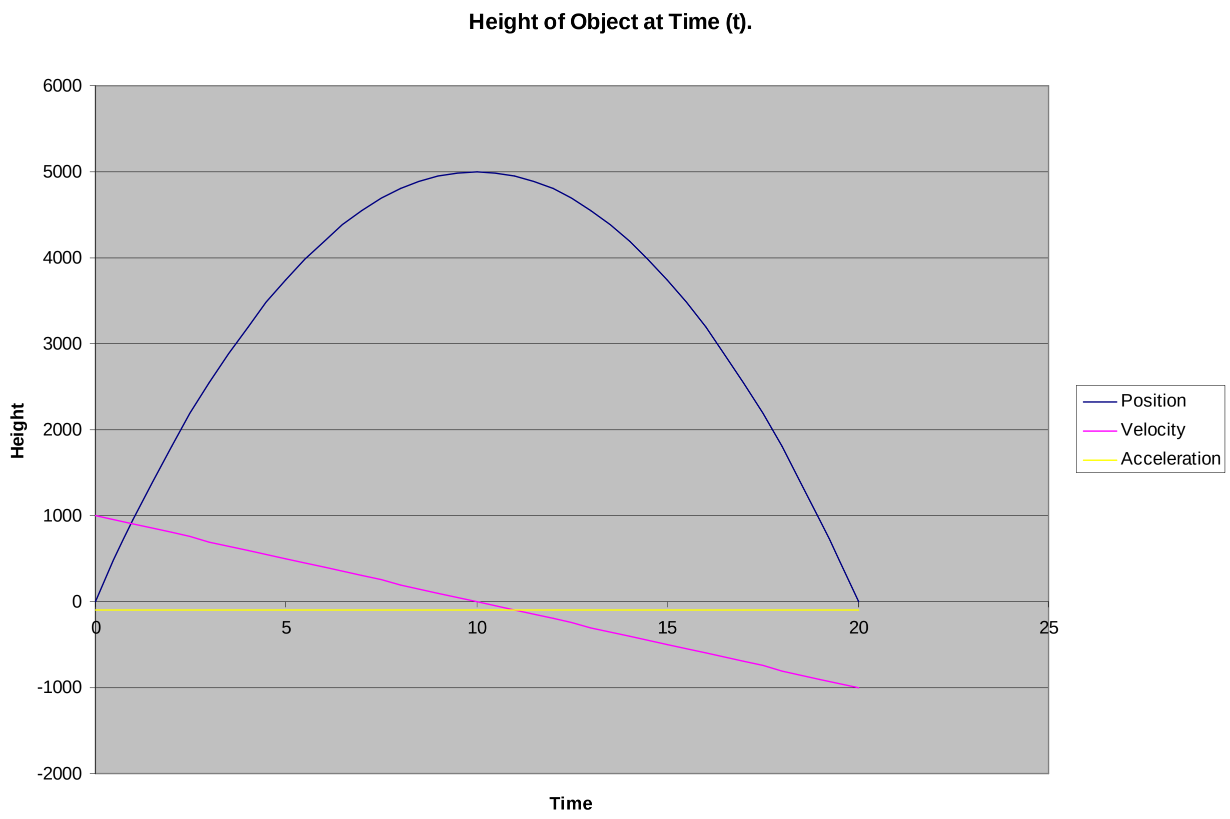
Consider a function that explains the position of a object at a point in time. The derivative of that function with respect to time describes the object's velocity. The second derivative describes the change in velocity over time, that is, the object's acceleration.

Examples: ICBM, drag racer.

Example of second derivative

Object shot straight up			
Time	Position	Velocity	Acceleration
0	0	1000	-100
1	950	900	-100
2	1800	800	-100
3	2550	700	-100
4	3200	600	-100
5	3750	500	-100
6	4200	400	-100
7	4550	300	-100
8	4800	200	-100
9	4950	100	-100
10	5000	0	-100
11	4950	-100	-100
12	4800	-200	-100
13	4550	-300	-100
14	4200	-400	-100
15	3750	-500	-100
16	3200	-600	-100
17	2550	-700	-100
18	1800	-800	-100
19	950	-900	-100
20	0	-1000	-100

Drawing of Example



Economic Example

Consider a function that describes the natural logarithm of the price level of an economy at any moment in time. The derivative of that function with respect to time represents the instantaneous inflation rate. The second derivative with respect to time represents the rate of change of inflation. During a hyperinflation, inflation accelerates for a period of time.

Production Theory

Consider the short run production function,

$$y = 100L^{0.7}$$

Marginal Product is

$$\mathrm{MPP_L} = \frac{dy}{dL} = 70L^{-0.3}$$

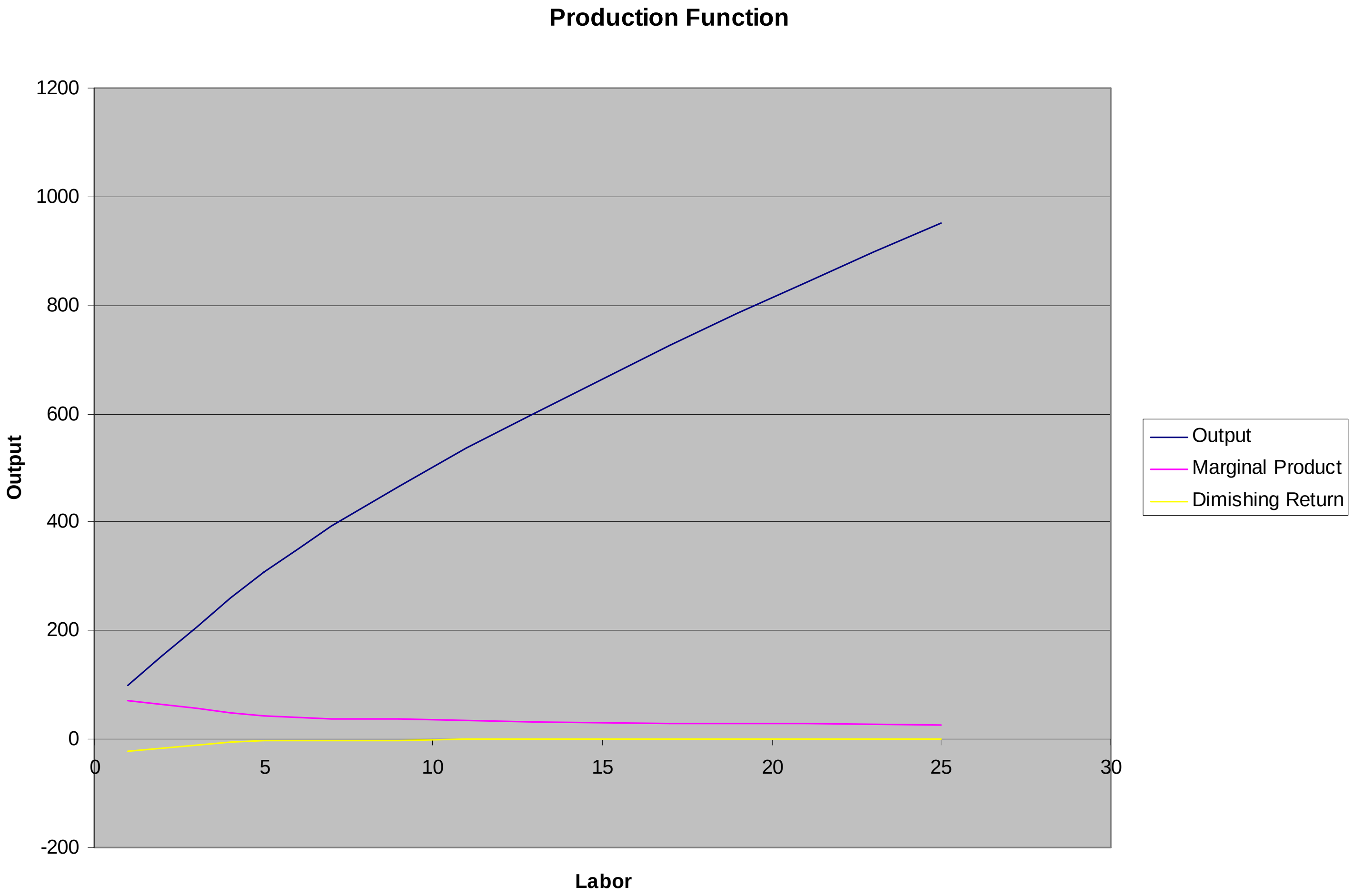
We see that the law of dimishing marginal product holds

$$\frac{\mathrm{dMPP_L}}{\mathrm{dL}} = \frac{d^2 y}{dL^2} = -21L^{-1.3}$$

Production Function

Labor	Output	Marginal Product	Dimishing Return
1	100	70	-21
5	309	43.19	-2.59
9	466	36.21	-1.21
13	602	32.43	-0.75
17	727	29.92	-0.53
21	842	28.08	-0.40
25	952	26.65	-0.32

Graph of our Production Function



Two Utility Functions

From microeconomics we know that more of a good is preferred to less of a good (**nonsatiation**). In addition, sometimes the assumption is made that the extra benefit from consuming an additional unit of a good is greater when overall consumption of the good is lower than when it is higher (**diminishing marginal utility**).

Two Utility Functions

Square Root Utility Function

$$U(c) = \alpha\sqrt{c}, \alpha > 0 \; and \; c > 0$$

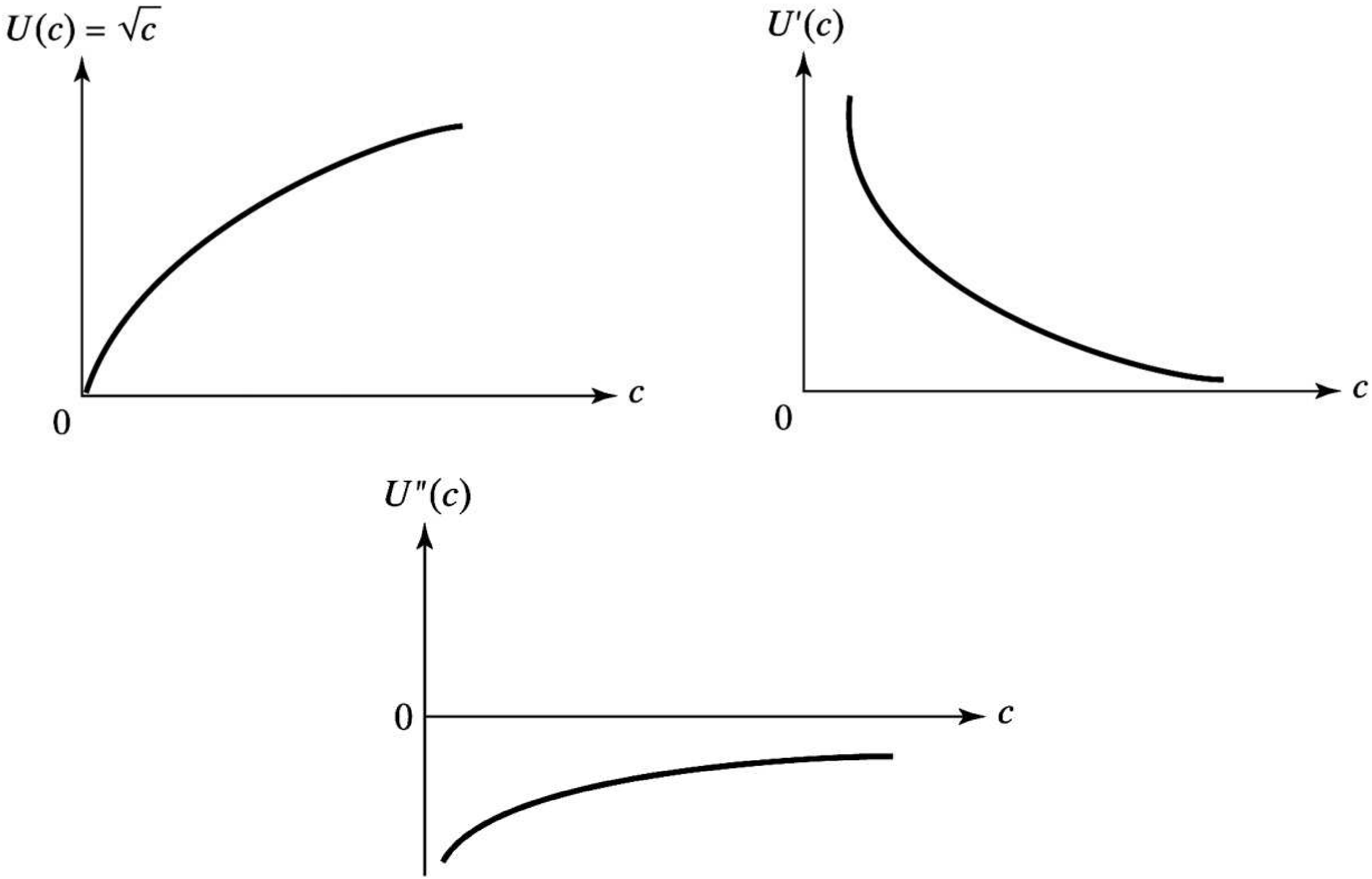
Marginal Utility

$$U'(c) = \frac{1}{2} \cdot \frac{\alpha}{\sqrt{c}} > 0 \Rightarrow nonsatiation$$

Dimishing Marginal Utility

$$U''(c) = -\frac{\alpha}{4} \cdot c^{-3/2} < 0$$

Figure 7.4 Square Root Utility Function



Two Utility Functions

Logarithmic Utility Function

$$U(c) = \beta \ln(c) \text{ where } \beta > 0 \text{ and c} > 0.$$

Marginal Utility

$$U'(c) = \frac{\beta}{c} > 0 \Rightarrow Nonsatiation$$

Dimishing Marginal Utility

$$U''(c) = -\frac{\beta}{c^2} < 0$$

Figure 7.5 Logarithmic Utility Function