Economics 2301

Lecture 24

Concavity

Two Utility Functions

From microeconomics we know that more of a good is preferred to less of a good (**nonsatiation**). In addition, sometimes the assumption is made that the extra benefit from consuming an additional unit of a good is greater when overall consumption of the good is lower than when it is higher (**diminishing marginal utility**).

Two Utility Functions

Square Root Utility Function

$$U(c) = \alpha\sqrt{c}, \alpha > 0 \; and \; c > 0$$

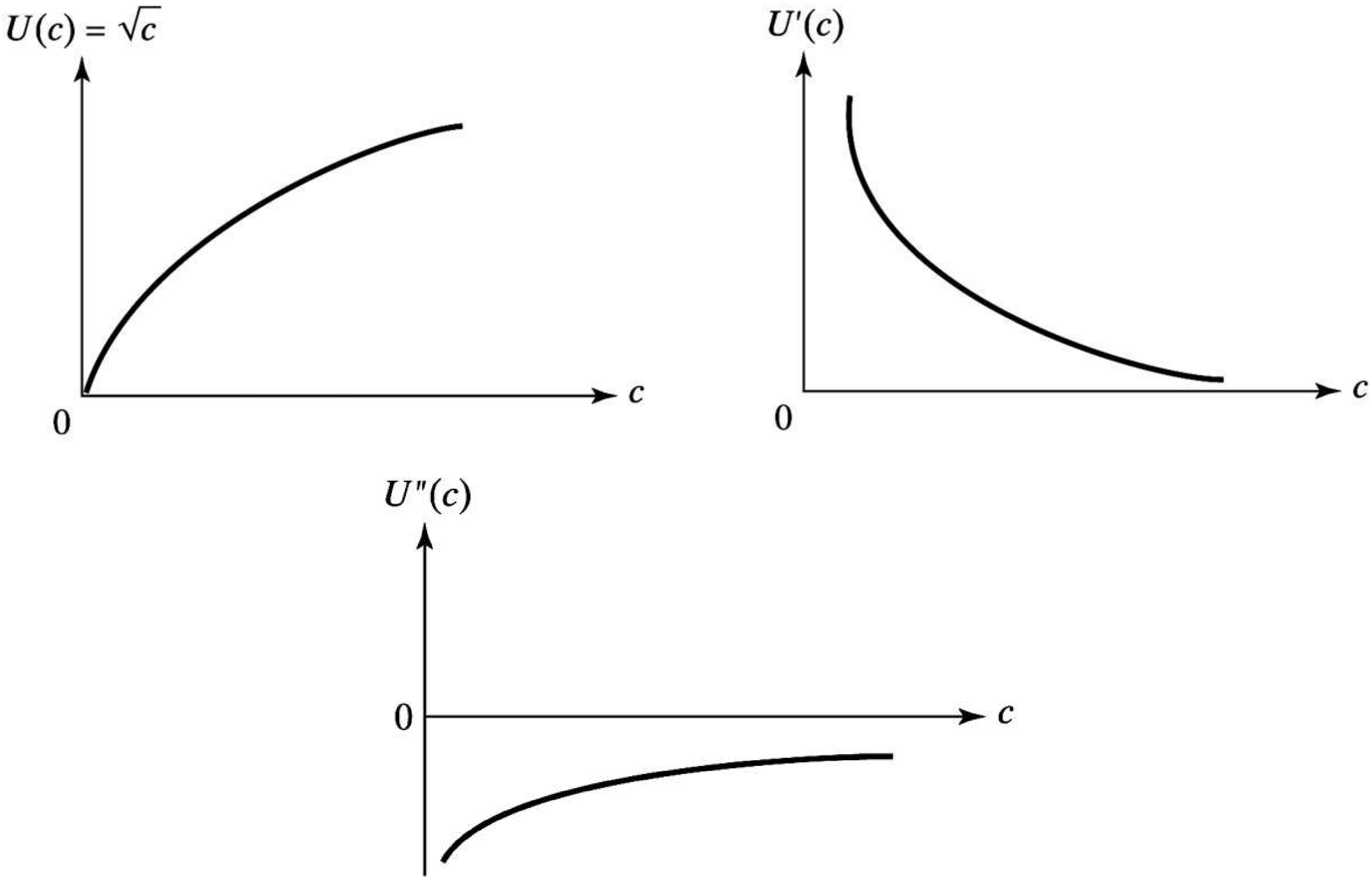
Marginal Utility

$$U'(c) = \frac{1}{2} \cdot \frac{\alpha}{\sqrt{c}} > 0 \Rightarrow nonsatiation$$

Dimishing Marginal Utility

$$U''(c) = -\frac{\alpha}{4} \cdot c^{-3/2} < 0$$

Figure 7.4 Square Root Utility Function



Two Utility Functions

Logarithmic Utility Function

$$U(c) = \beta \ln(c) \text{ where } \beta > 0 \text{ and c} > 0.$$

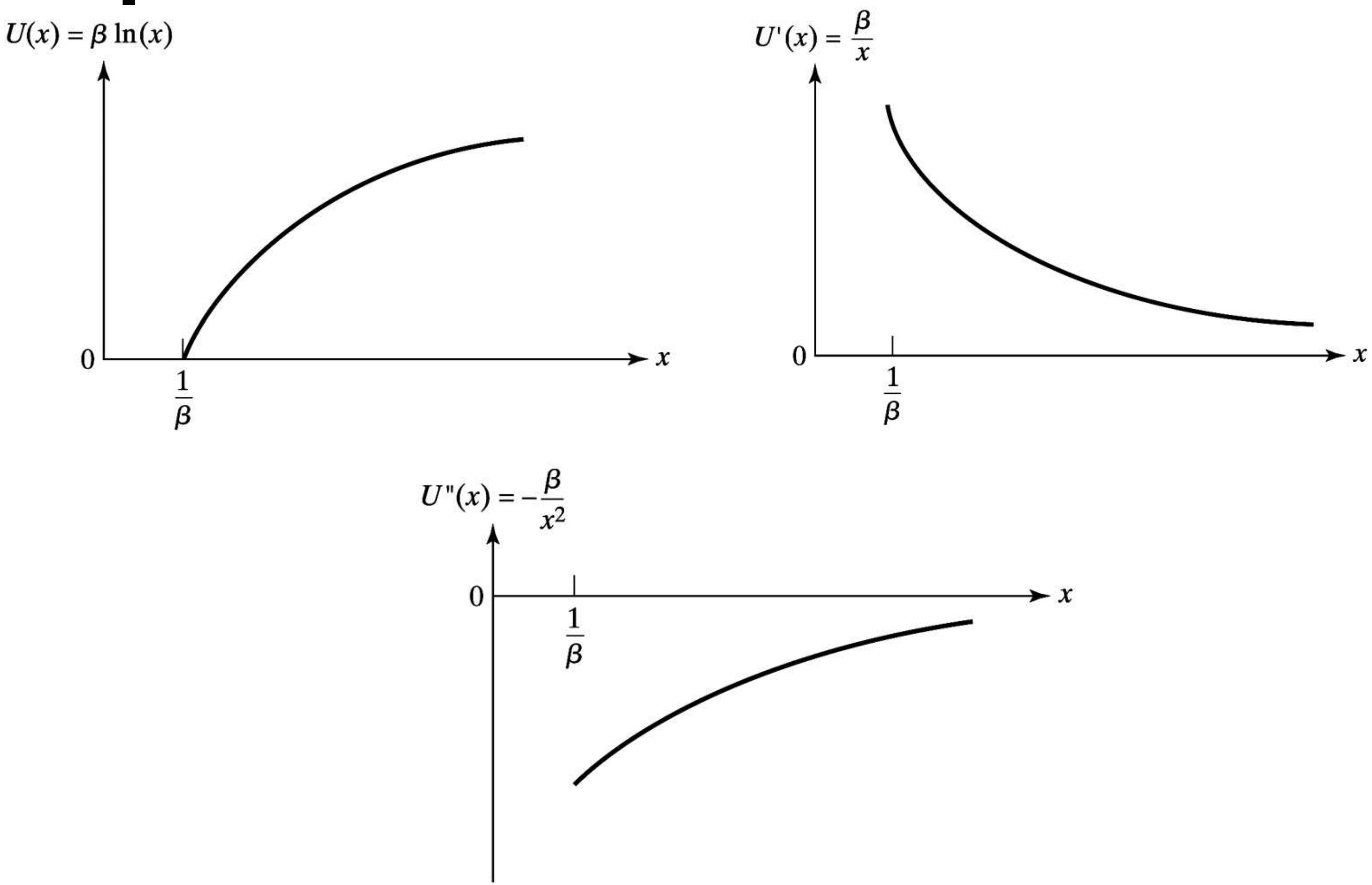
Marginal Utility

$$U'(c) = \frac{\beta}{c} > 0 \Rightarrow Nonsatiation$$

Dimishing Marginal Utility

$$U''(c) = -\frac{\beta}{c^2} < 0$$

Figure 7.5 Logarithmic Utility Function



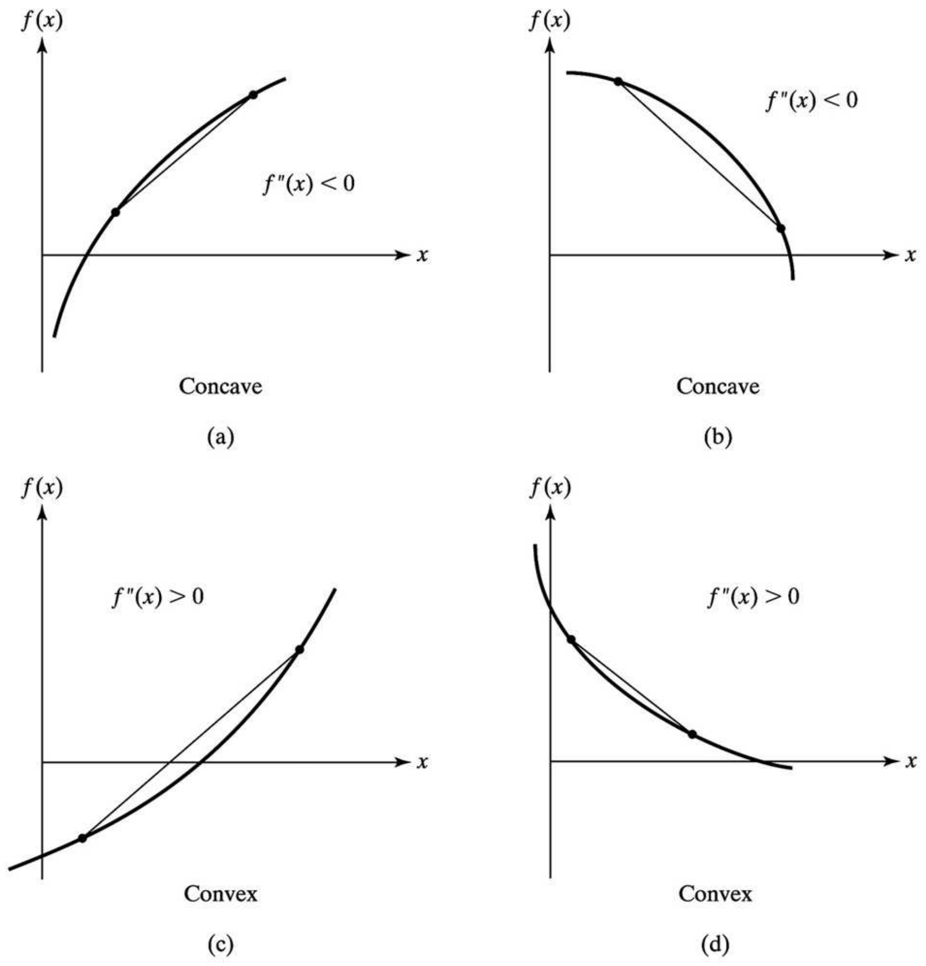
Concavity

The concavity of a function used in economics often corresponds to an important economic concept. For example, the concavity of a utility function which illustrates how marginal utility changes with consumption, reflects the concept of diminishing marginal utility. Likewise, the concavity of a production function, which shows how marginal product declines with the increased use of an input, is the mathematical counterpart to the economic concept of diminishing returns to inputs.

Concavity

- Concavity of a function reflects its curvature.
- A function is strictly convex over an interval if a secant line connecting any two points in that interval lies wholly above the graph of the function.
- A function is strictly concave over an interval if a secant line connecting any two points in that interval lies wholly below the graph of the function.

Figure 7.6 Concave and Convex Functions



Strictly Concave and Convex

A function $f(x)$ is stricly concave over an interval if $f''(x) < 0$ for all values of x in that interval.

A function $f(x)$ is strictly convex over an interval if $f''(x) > 0$ for all values of x in that interval.

Concave and Convex

A function $f(x)$ is concave over an interval if $f''(x) \leq 0$ for all values of x in that interval.

A function $f(x)$ is concex over an interval if $f''(x) \geq 0$ for all values of x in that interval