# Economics 2301

Lecture 25

Risk Aversion

# Risk Aversion

The curvature of a utility function is related to its implicit modeling of risk aversion.

Game: You can have \$10 for sure, or get \$0 if when you cut a deck of cards you get less than an ace, or receive \$130 if you cut an ace.

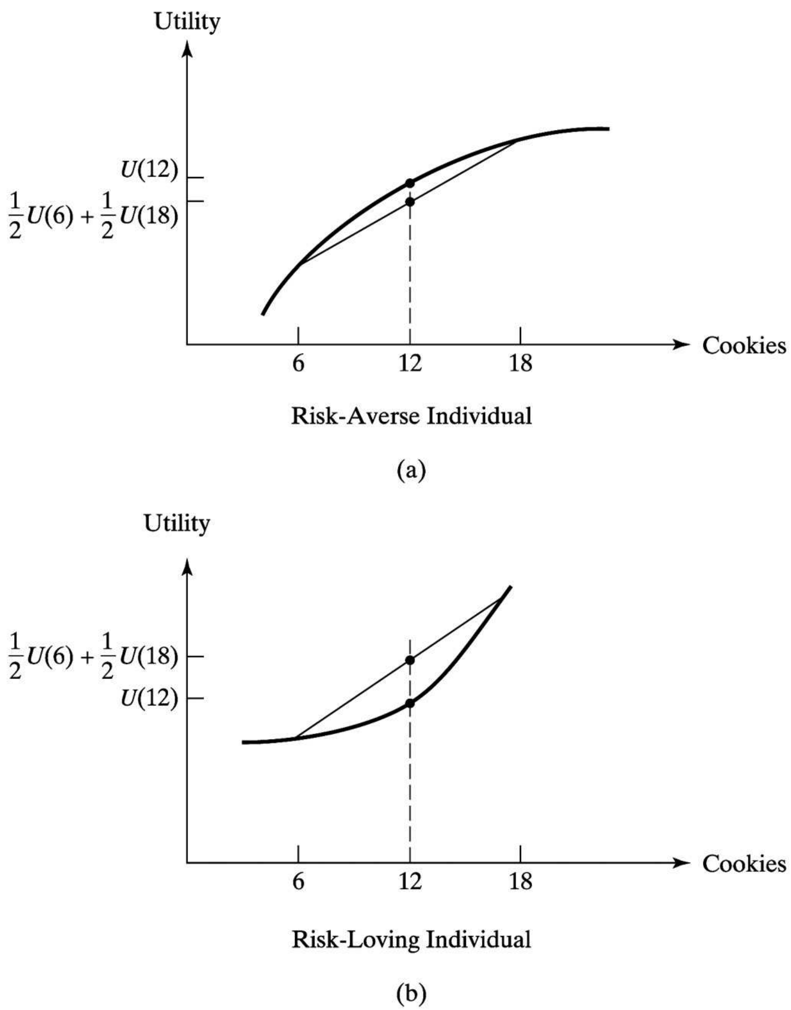
$$E(\text{return}) = \frac{12}{13}\cdot(\$0) + \frac{1}{13}\cdot(\$130) = \$0 + \$10 = \$10$$

This is a fair game.

If E(Utility of the Game) > Utility(sure thing), one is risk lover. If E(Utility of the Game) < Utility (sure thing) one is risk adverse.

$$E(\text{Utility of Game}) = P(\text{Loss})\cdot \text{Utility(loss)} + P(\text{win})\cdot \text{Utility(win)}$$

# Figure 7.7 Utility Functions for Risk-Averse and Risk-Loving Individuals



# Example-Risk Adverse

Risk Adverse: Assume you have the following square root utility function: $U(m) = 5 \cdot \sqrt{m}$

Utility sure thing: $U(\$10) = 5 \cdot \sqrt{10} = 5 \cdot (3.1623) = 15.81$

Expected Utility of gamble: $\frac{12}{13} \cdot U(0) + \frac{1}{13} \cdot U(130)$

$$= \frac{12}{13} \cdot 5\sqrt{0} + \frac{1}{13} \cdot 5 \cdot \sqrt{130} = 0 + \frac{5 \cdot 11.402}{13} = 4.39 < 15.81$$

Person will select the sure thing because it maximizes their expected utility and they are risk adverse.

# Proof of Risk Adverse

$$U(m) = 5 \cdot \sqrt{m}$$

$$U'(m) = \frac{5}{2} \cdot (m)^{-\frac{1}{2}}$$

$$U''(m) = -\frac{5}{4} \cdot (m)^{-\frac{3}{2}} < 0$$

The utility function is concave and hence risk adverse.

# Example-Risk Lover

For our risk lover assume the individual has the following utility function: $U(m) = 5 \cdot m^{1.2}$

Utility sure thing: $U(10) = 5 \cdot (10)^{1.2} = 79.24$

Utility of the gamble: $E(U(m)) = \frac{12}{13} \cdot 5 \cdot (0)^{1.2} + \frac{1}{13} \cdot 5 \cdot (130)^{1.2}$

$$= 0 + \frac{5 \cdot 344.1375}{13} = 132.36 > 79.24$$

$\therefore$ Our risk lover selected the gamble as it maximizes his expected utility.

# Proof Risk Lover

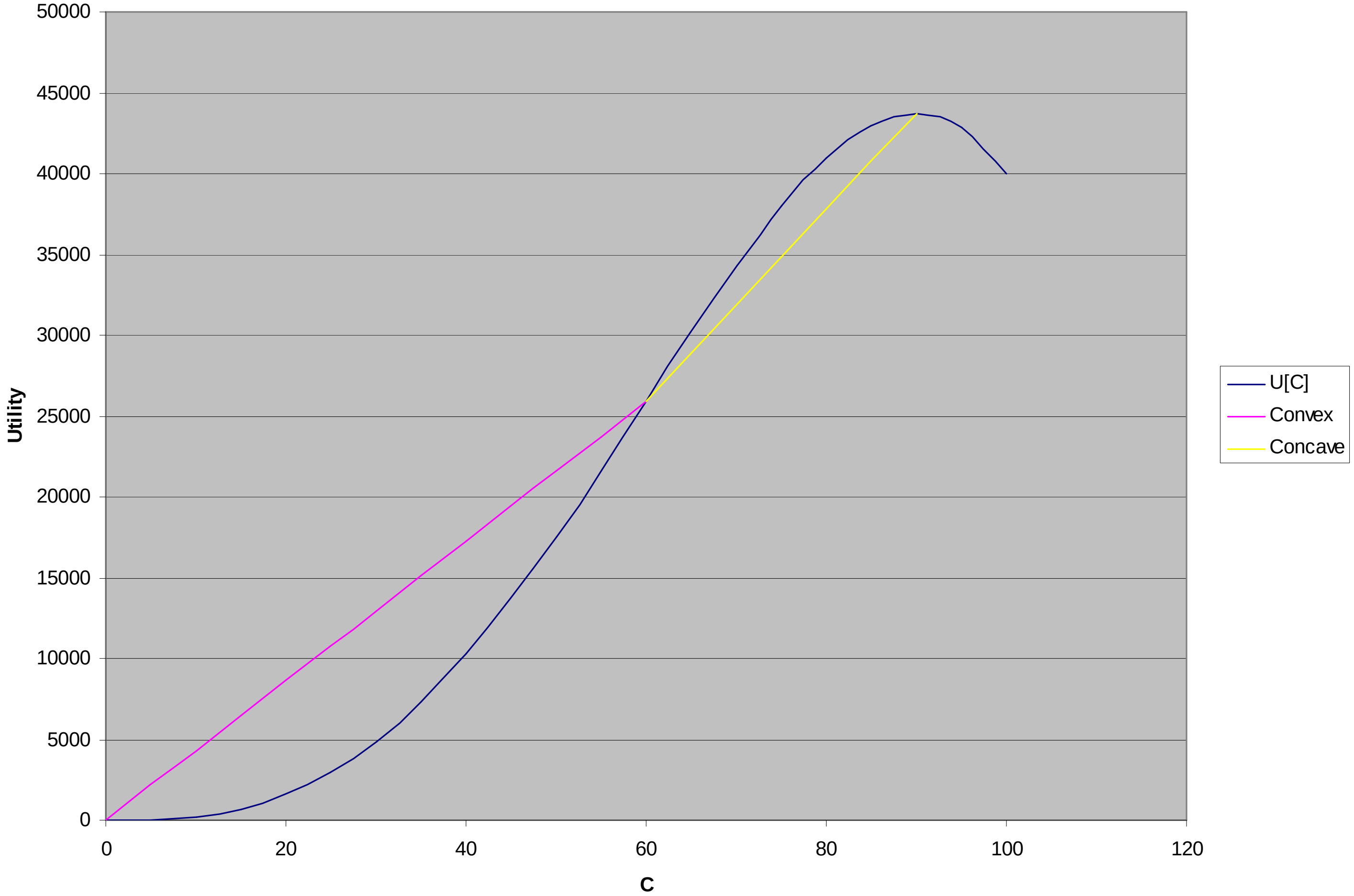
$$U(m) = 5 \cdot (m)^{1.2}$$

$$U'(m) = 6 \cdot (m)^{.2}$$

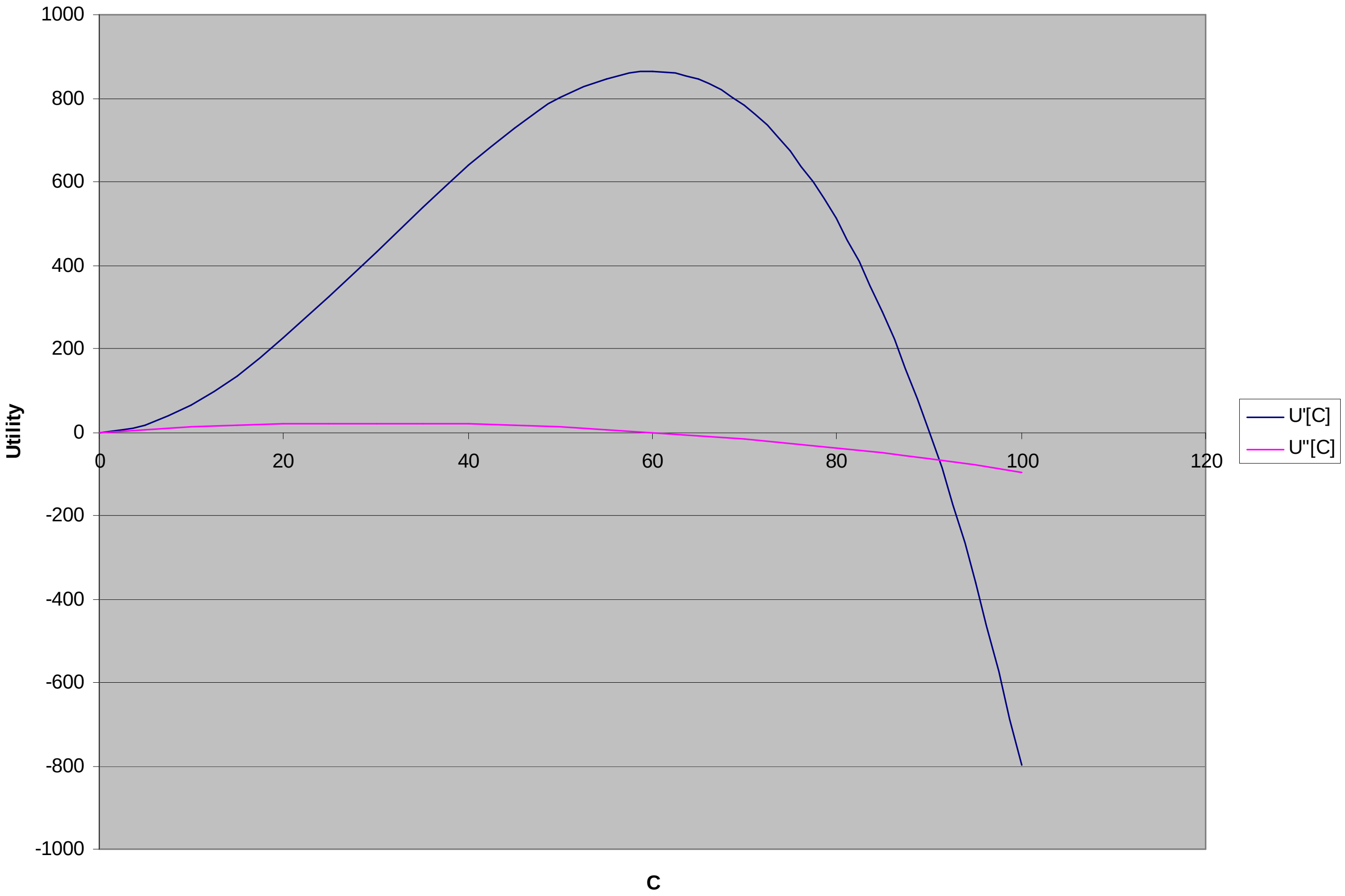
$$U''(m) = 1.2 \cdot (m)^{-.8} > 0$$

Function is convex and the person is a risk lover.

# Person who is both risk-lover and risk-adverse



# Plot of Marginal Utility and Second Derivative



# Proof of Properties of both risk-lover and risk-adverse

$$U(C) = 0.24 \cdot C^3 - 0.002 \cdot C^4$$

$$U'(C) = 0.72 \cdot C^2 - 0.008C^3$$

$$Roots = 0{,}90$$

$$U''(C) = 1.44 \cdot C - 0.024C^2$$

$$Roots = 0{,}60$$

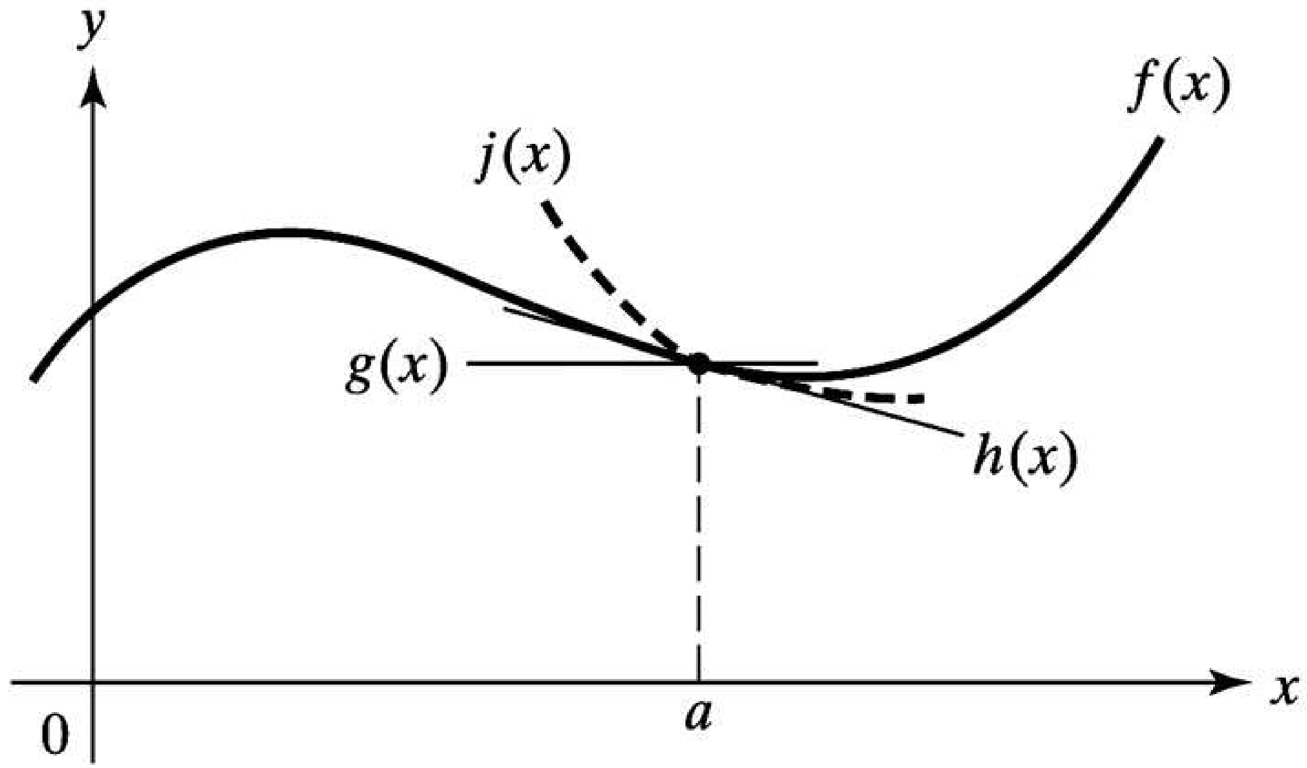
Our orginal function maximum at 90 and inflection point at 0 and 60.

Function is convex from 0 to 60 and concave beyond 60

# Linearization of Functions and Taylor Series

- It is often useful to be able to express a function in linear form.
- Helpful if we want to use matrix algebra or simple econometrics.
- Calculus can give us a linear approximation to a function.

# Figure 7.8 Taylor Series Approximations



# Linearization

We have the function $y = f(x)$ that we wish to approximate by a linear function at the point $a$. The simpliest approximation is $g(x) = f(a)$. Not good when we get any distance from point.

We prefer a linear function of the form

$h(x) = f(a) + b \cdot (x\text{-}a)$.

Question? How do we select "$b$?".

The differential is the equation for the tangent line to the function at the point $x = a$. Therefore the best linear approximation is $h(x) = f(a) + f'(a)(x\text{-}a)$.

# Quadratic Approximation

A better approximation allows for some curvature.

Quadratic approximation would be

$j(x) = f(a) + f'(a)(x-a) + c(x-a)^2.$

Want $c$ related to the second derivative of the function at $x = a$.

*Want* $j''(a) = f''(a).$

$J''(a) = 2c = f''(a) \Rightarrow c = f''(a)/2$

$\therefore\ j(x) = f(a) + f'(a)(x-a) + \frac{1}{2} \cdot f''(a) \cdot (x-a)^2.$

# Nth Degree Taylor Expansion

The $n^{th}$ degree approximation to our function at $x = a$ is

$$m(x) = \frac{f(a)}{0!} + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

*Where*

$f^{(n)}(a) =$ nth derivative of $f(x)$ evaluated at $x = a$.

$$n! = n \cdot (n\text{-}1) \cdot (n-2) \cdot \cdots \cdot 2 \cdot 1$$

$$0! = 1$$

$$1! = 1$$

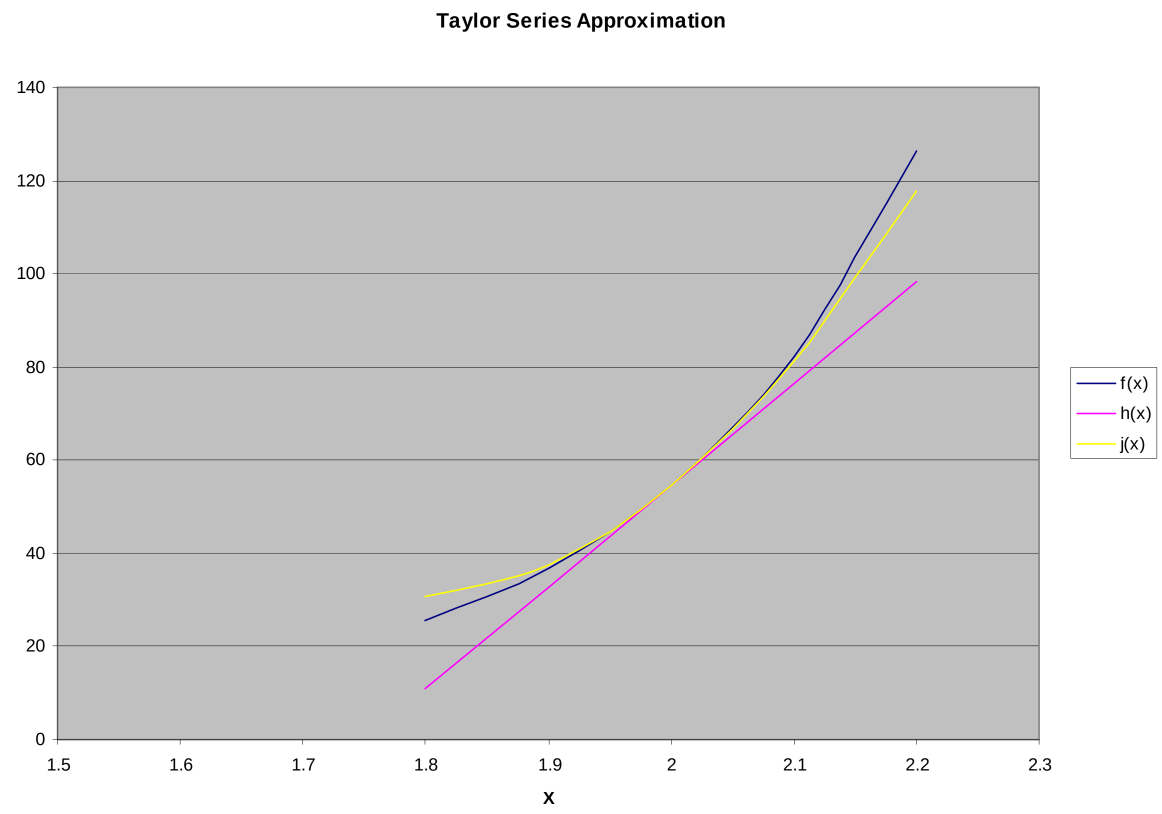
# Example

$$Consider\ y = e^{x^2} \text{ at the point } x = 2.$$

$$\text{Linear}: h(x) = e^4 + 4e^4 \cdot (x-2) = -7e^4 + 4e^4 \cdot x$$

$$\text{quadratic}: j(x) = e^4 + 4e^4 \cdot (x-2) + 9e^4 \cdot (x-2)^2$$

# Graph of our approximations



# Accuracy of our approximation

| x | f(x) | h(x) | j(x) | Linear % Error | Quadratic % Error |
|---|---|---|---|---|---|
| 1.8 | 25.53372 | 10.91963 | 30.57496 | -57% | 20% |
| 1.9 | 36.96605 | 32.75889 | 37.67272 | -11% | 2% |
| 2 | 54.59815 | 54.59815 | 54.59815 | 0% | 0% |
| 2.1 | 82.26946 | 76.43741 | 81.35124 | -7% | -1% |
| 2.2 | 126.4694 | 98.27667 | 117.932 | -22% | -7% |