Economics 2301

Lecture 7

Exponential Functions

Present Value

- **Present value** represents the amount required today to yield a certain payout at a given time in the future.

Present Value

Present value wit Continuous Compounding

$$P(t) = X(t+n)e^{-rn}$$

Present Value with Discrete Compounding

$$P_t = X_{t+n}\left(1+r\right)^{-n}$$

Zero Coupon Bond

A zero coupon bond, on the other hand, is sold at a discount from its face value and the issuer makes no interest payments during the life of the security. When it matures, you receive the full face amount which equals your initial investment plus accumulated interest compounded over the life of the bond. (The examples cited refer to issues sold in primary market offerings.)

For example, an investor could purchase a 20-year municipal zero coupon bond with a face amount of $20,000 for approximately $6,757. When the bond matures, the investor receives the full, face amount, $20,000. The $13,243 difference is attributable to the accumulated compounded interest, in this case calculated on the basis of a 5.5% rate of return.

Price Zero Coupon Bond

Consider a zero coupon bond with 12 year maturity and 5% rate. Face of bond is $10,000

Continuous Compounding

$$P(t) = X(t+n)e^{-rn} = \$10{,}000 * e^{-.05*12}$$

$$= \$10{,}000 * e^{-.6} = \frac{\$10{,}000}{1.822} = \$5488.12$$

Discrete Compounding

$$P_t = X_{t+n}(1+r)^{-n} = \$10{,}000 * (1.05)^{-12}$$

$$= \frac{\$10{,}000}{1.7959} = \$5{,}568.37$$

Pricing Zero Coupon Bonds

	Price of Zero Coupon Bond					
Rate:	3%		5%		7%	
Maturity in years	Continuous	Discrete	Continuous	Discrete	Continuous	Discrete
8	$7,866.28	$7,894.09	$6,703.20	$6,768.39	$5,712.09	$5,820.09
12	$6,976.76	$7,013.80	$5,488.12	$5,568.37	$4,317.11	$4,440.12
16	$6,187.83	$6,231.67	$4,493.29	$4,581.12	$3,262.80	$3,387.35
20	$5,488.12	$5,536.76	$3,678.79	$3,768.89	$2,465.97	$2,584.19

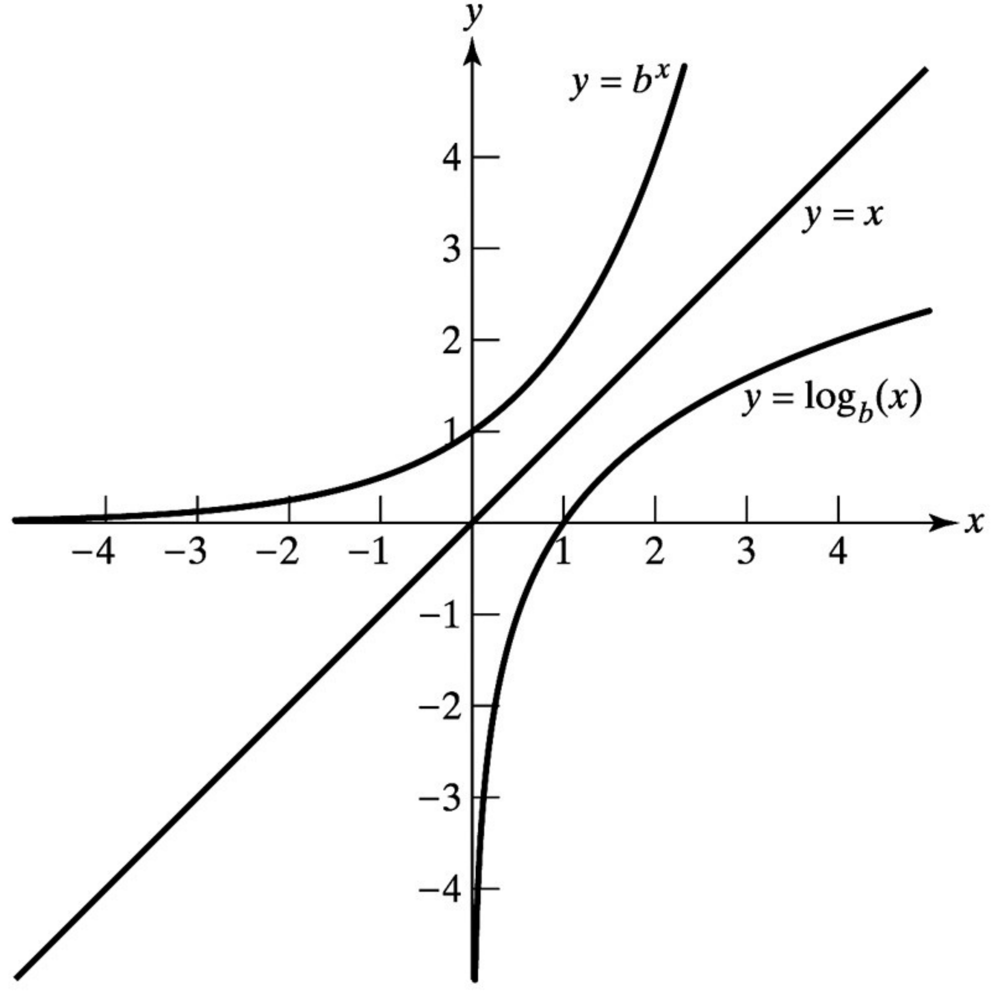
Logarithmic Functions

- Any exponential function has an inverse since it is strictly monotonic and , therefore, one-to-one.
- The class of functions that are inverses to exponential functions are **logarithmic functions.**

Logarithmic Functions

- Any point (i,j) in the exponential function $y=b^x$ has a corresponding point (j,i) in the logarithmic function $y=\log_b(x)$.
- Like the exponential function, the logarithmic function is strictly monotonic and increasing.

Figure 3.3 An Exponential Function and Its Inverse



Properties of logarithmic functions

- Logarithmic functions are everywhere concave while exponential functions are everywhere convex.
- The arguments of a logarithmic function are restricted to the set of positive real numbers, while the range is the set of all real numbers. The converse is the case of the exponential function.
- Since exponential functions cross the y-axis at (0,1), logarithmic functions cross the x-axis at (1,0).

Logarithmic Function with Base b

The logarithmic function $y = \log_b(x)$ which is read as "y is the base b logarithm of x," satisfies the relationship $b^y = x.$

This definition of lagarithms implies $\log_b(b) = 1$ for any base since $b^1 = b.$ By the definition of an inverse function, we also have $b^{\log_b(x)} = x.$