Economics 2301

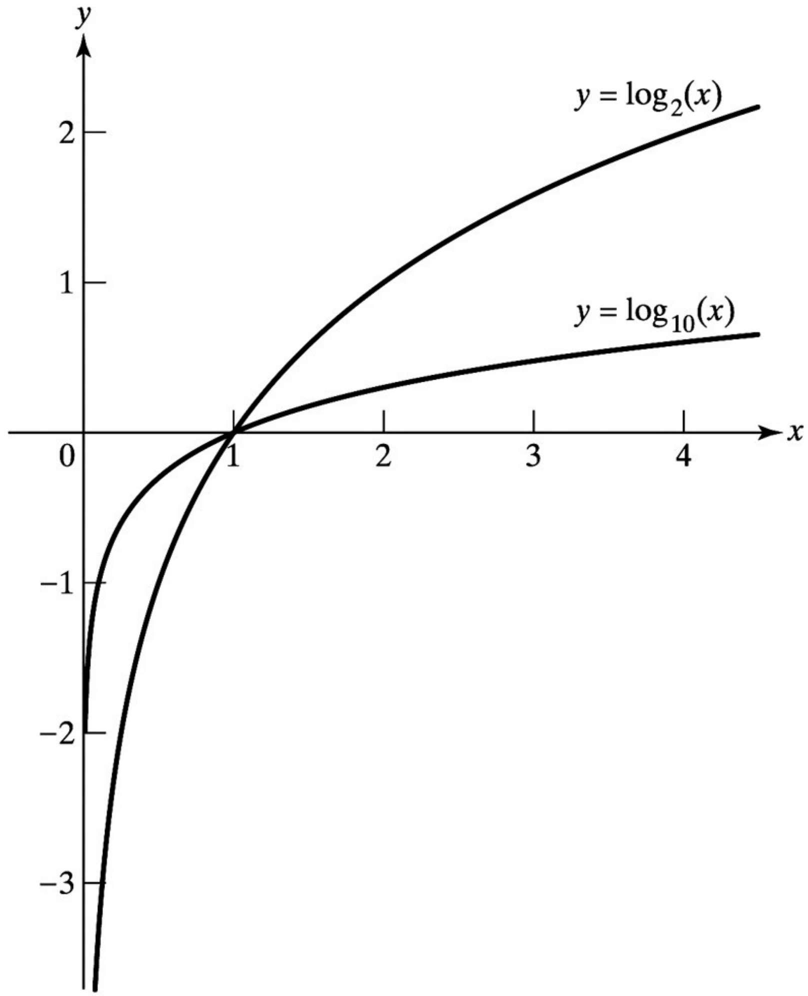
Lecture 8

Logarithms

Base 2 and Base 10 Logarithms

Base 2 Logarithms		Base 10 Logarithms	
$Log_2(0.25)=-2$	since $2^{-2}=1/4$	$Log_{10}(0.01)=-2$	since $10^{-2}=1/100$
$Log_2(0.5)=-1$	since $2^{-1}=1/2$	$Log_{10}(0.1)=-1$	since $10^{-1}=1/10$
$Log_2(1)=0$	since $2^0=1$	$Log_{10}(1)=0$	since $10^0=1$
$Log_2(2)=1$	since $2^1=2$	$Log_{10}(10)=1$	since $10^1=10$
$Log_2(4)=2$	since $2^2=4$	$Log_{10}(100)=2$	since $10^2=100$
$Log_2(8)=3$	since $2^3=8$	$Log_{10}(1000)=3$	since $10^3=1000$

Figure 3.4 Base 2 and Base 10 Logarithms



Rules of logarithmic transformations

Product

$$\log_b(XY) = \log_b(X) + \log_b(Y)$$

Quotient

$$\log_b(X/Y) = \log_b(X) - \log_b(Y)$$

Exponent

$$\log_b(X^\lambda) = \lambda \log_b(X)$$

Relationship between logarithms with different bases

Let b and c be bases for two sets of logarithms.

$$\log_b(x) = \log_b(c^{\log_c(x)}) = \log_c(x)\log_b(c)$$

or

$$\log_b(c) = \frac{\log_b(x)}{\log_c(x)}$$

$$if\;\; b < c \Rightarrow \log_b(c) > 1$$

$$\Rightarrow \frac{\log_b(x)}{\log_c(x)} > 1$$

$$\Rightarrow |\log_b(x)| > |\log_c(x)|$$

Key Transformation

Given the results from the previous slide

$$\log_{10}(x) = \log_{10}(e)\log_e(x) = 0.4343 * \log_e(x)$$

$$\log_e(x) = \log_e(10)\log_{10}(x) = 2.3026 * \log_{10}(x)$$

Natural Logarithms

- A **natural logarithm** has as it base the exponential, e.
- We write natural logarithms of x as $\log_e(x)$ or $\ln(x)$.
- Natural logarithms have many applications in economics

Properties of natural logarithms

$$* \ln(e^z) = z$$

$$* e^{\ln X} = X$$

$$* \ln(XY) = \ln(X) + Ln(Y)$$

$$* \ln\left(\frac{X}{Y}\right) = \ln(X) - \ln(Y)$$

$$\ln\left(X^Z\right) = Z \ln X$$

Rule of 70

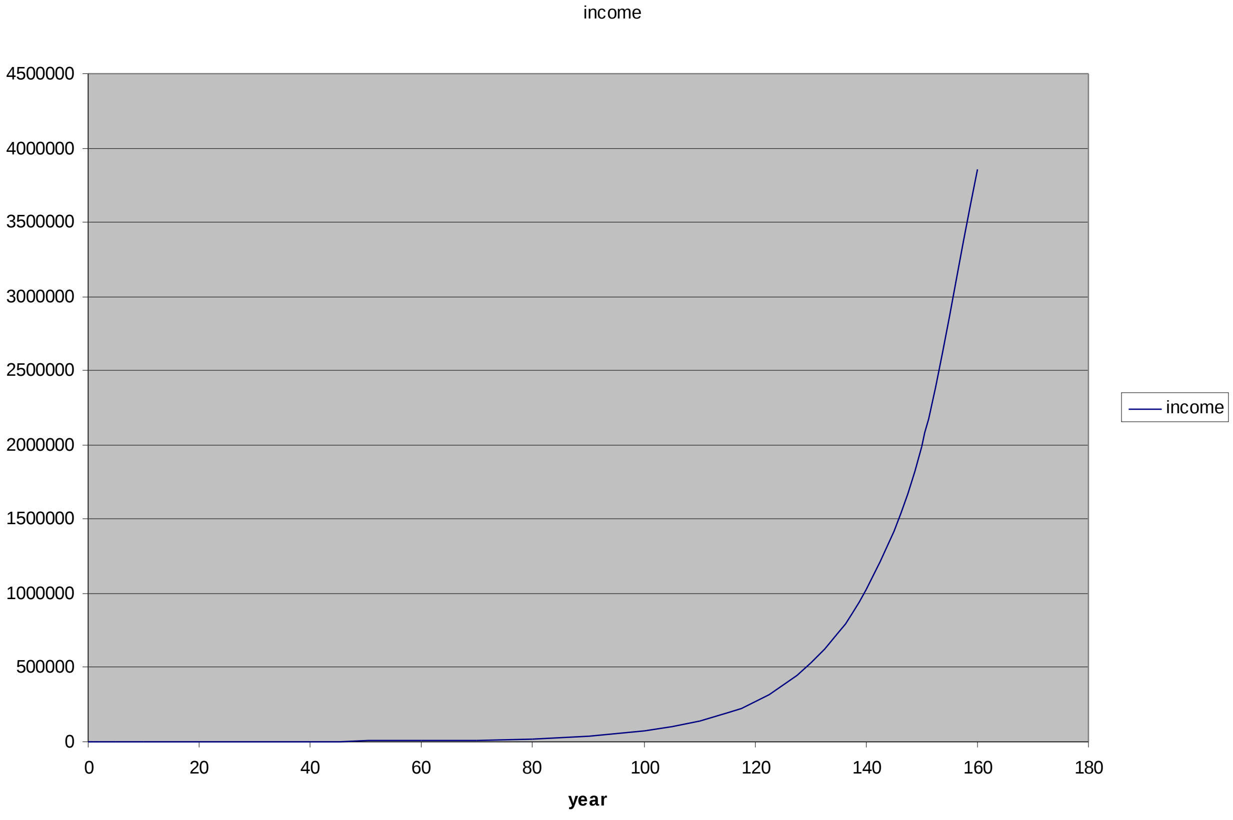
How long does it take for your money to double at interest rate r with continuous compound. A good approximation is

$$n = \frac{70}{r}$$ where r equal interest rate in percentage.

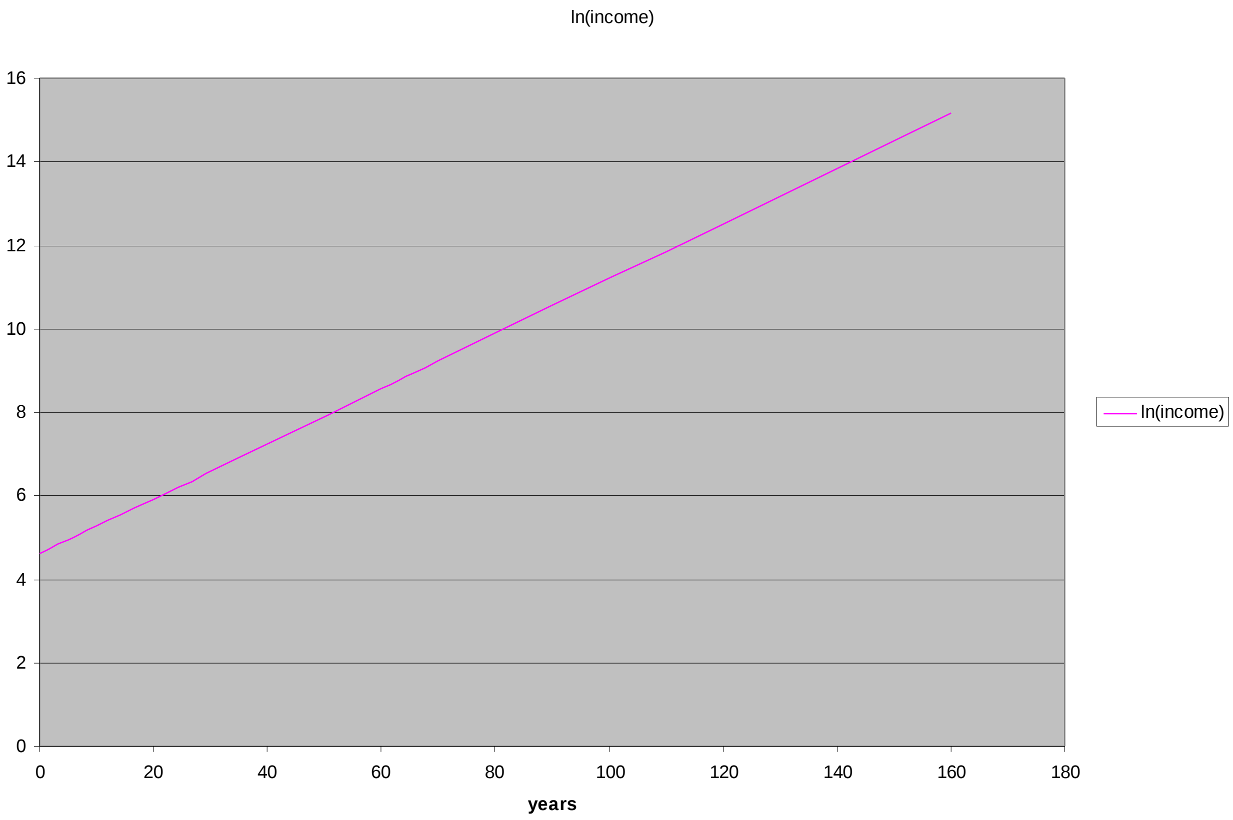
For simple compounding the rules is 72

$$n = \frac{72}{r}$$

Graphing Grow Data



Graphing Log of income



The log Transformation

$$Suppose\ X(t+n) = X(t)e^{rn}$$

$$then\ \ln(X(t+n)) = \ln\left(X(t)\right) + rn$$

This is a linear line with slope r, intercept

$\ln\left(X(t)\right)$ and independent variable n.