Definitions or Equations (10 pts)

Transpose of the product of two conformable matrices

\[(AB)' = B'A'\]

The inverse of the product of two nonsingular matrices A and B.

\[(AB)^{-1} = B^{-1}A^{-1}\]

The inner product of the two nx1 vectors x and y.

The inner product of the two vectors x and y is

\[x'y = y'x = \sum_{i=1}^{n} x_i y_i\]

General formula for the inverse of the nonsingular square matrix A.

\[A^{-1} = \frac{1}{|A|} C' \quad C = \text{cofactor matrix for } A.\]

Cramer's Rule

For the equation system \(Ax = c\), (A is nonsingular), the solution for the \(i\)th element of x is

\[x_i = \frac{|A_i|}{|A|}\]

where \(A_i\) is a matrix with the \(i\)th column of A replaced by c.
Problems (10 pts)

Use repeated substitution (and no programmable calculator) to solve for the equilibrium values of \( w \), \( x \) and \( y \) in the following system of equations.

\[
\begin{align*}
y &= 2w + 8 \\
x &= 4w - 6 \\
y &= 2x + 2
\end{align*}
\]

Using first equation, last equation
\[
\begin{align*}
2x + 2 &= 2w + 8 \\
2x &= 2w + 6 \\
x &= w + 3
\end{align*}
\]

using equation 2, we get
\[
\begin{align*}
w + 3 &= 4w - 6 \\
9 &= 3w \\
w &= 3 \\
x &= w + 3 = 3 + 3 = 6 \\
y &= 2x + 2 = 2(6) + 2 = 12 + 2 = 14
\end{align*}
\]

You are given the matrix \( A \) below:

\[
A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 4 & 0 \\ 3 & 2 & 1 \end{bmatrix}
\]

a) Calculate the determinant of \( A \) (no programmable calculators).

\[
C = \begin{bmatrix} 4 & 0 & -12 \\ -1 & -1 & 5 \\ -4 & 0 & 8 \end{bmatrix}
\]

Expanding on the second row of \( A \), we get the determinant

\[
|A| = \sum_{j=1}^{3} a_{3j} c_{2j} = 0(-1) + 4(-1) + 0(5) = -4
\]

b) Find the inverse matrix of \( A \) and prove it is the inverse (no programmable calculators).
$$A^{-1} = \frac{1}{|A|} \begin{pmatrix} 4 & -1 & -4 \\ 0 & -1 & 0 \\ -12 & 5 & 8 \end{pmatrix} \begin{pmatrix} -1 & 1/4 & 1 \\ 0 & 1/4 & 0 \\ 3 & -5/4 & -2 \end{pmatrix}$$

$$A^{-1} A = \begin{pmatrix} -1 & 1/4 & 1 \\ 0 & 1/4 & 0 \\ 3 & -5/4 & -2 \end{pmatrix} \begin{pmatrix} 2 & 3 & 1 \\ 0 & 4 & 0 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$