Economics 2301 **Name**____
Quiz 3 **Fall, 2010**

Definitions or Equations (10 pts)

Transpose of the product of two conformable matrices

$$(AB)'=B'A'$$

The inverse of the product of two nonsingular matrices A and B.

$$(AB)^{-1}=B^{-1}A^{-1}$$

The inner product of the two nx1 vectors **x** and **y**.

The inner product of the two vectors **x** and **y** is $x'y=y'x=\sum_{i=1}^{n} x_i y_i$

General formula for the inverse of the nonsingular square matrix **A**.

$A^{-1}=\frac{1}{|A|}C'$ **C** = cofactor matrix for **A**.

Cramer's Rule

For the equation system **Ax=c,** (**A** is nonsigular), the solution for the i*th* element of **x** is

$x_i=\frac{|A_I|}{|A|}$ where A_i is a matrix with the i*th* column of **A** replaced by **c**.

Problems (10 pts)

Use repeated substitution (and no programmable calculator) to solve for the equilibrium values of w, *x* and *y* in the following system of equations.

$y = 2w + 8$

$x = 4w - 6$

$y = 2x + 2$

$$\text{Using first equation, last equation}$$
$$2x + 2 = 2w + 8$$
$$2x = 2w + 6$$
$$x = ¿\, w + 3$$
$$\text{using equation } 2, \text{we get}$$
$$w + 3 = 4w - 6$$
$$9 = 3w$$
$$w = 3$$
$$x = w + 3 = 3 + 3 = 6$$
$$y = 2x + 2 = 2(6) + 2 = 12 + 2 = 14$$

You are given the matrix **A** below:

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 4 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

a) Calculate the determinant of A (no programmable calculators).

$$C = \begin{bmatrix} 4 & 0 & -12 \\ -1 & -1 & 5 \\ -4 & 0 & 8 \end{bmatrix}$$

. Expanding on the second row of **A**, we get the determinant

$$|A| = \sum_{j=1}^{3} a_{2j} c_{2j} = 0(-1) + 4(-1) + 0(5) = -4$$

b) Find the inverse matrix of A and prove it is the inverse (no programmable calculators).

$$A^{-1} = \frac{1}{|A|} C' = \frac{1}{(-4)} \begin{bmatrix} 4 & -1 & -4 \\ 0 & -1 & 0 \\ -12 & 5 & 8 \end{bmatrix} = \begin{bmatrix} -1 & 1/4 & 1 \\ 0 & 1/4 & 0 \\ 3 & -5/4 & -2 \end{bmatrix}$$

$$A^{-1} A = \begin{bmatrix} -1 & 1/4 & 1 \\ 0 & 1/4 & 0 \\ 3 & -5/4 & -2 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 0 & 4 & 0 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$