Economics 2301 **Name**________________________________

Quiz 5 **Fall 2010**

Definition or Equation (10 pts)

Partial derivative of function *y=f(x,z)* with respect to *x*.

$$\frac{\partial y}{\partial x} = \underset{\Delta x \to 0}{Lim} \frac{f(x+\Delta x, z) - f(x,z)}{\Delta x}$$ if it exists.

The n^{th} order Taylor series expansion of the function *y=f(x)* around the point *a*.

$$y \cong f(a) + \frac{f^{(1)}}{1!}(x-a) + \frac{f^{(2)}}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}}{n!}(x-a)^n$$

Q.E.D.

Latin Translation: quod erat demonstrandum

Mathematical meaning: You have successfully completed the proof you were to prove.

Young's Theorem.

For the multivariate function $f(x_1, \cdots, x_i, \cdots, x_j, \cdots, x_n)$

$$f_{ij}(x_1, \cdots, x_i, \cdots, x_j, \cdots, x_n) = f_{ji}(x_1, \cdots, x_i, \cdots, x_j, \cdots, x_n)$$

Differentiation rule for a function to be strictly convex over a defined interval

A function is strictly convex over an interval if *f"(x)>0* for all values of *x* in that interval.

Problems (10 pts)

Find the partial derivates of y with respect to x and z.

$$y = \left[e^{x^a}\right]^{\ln(Z)}$$

$$\frac{\partial y}{\partial x} = ax^{\alpha-1}\ln(z)e^{\ln(z)\cdot x^a} = ax^{\alpha-1}\ln(z)(e^{\cdot x^a})^{\ln(z)}$$

$$\frac{\partial y}{\partial z} = \frac{x^a}{z}e^{\ln(z)\cdot x^a} = \frac{x^a}{z}(e^{\cdot x^a})^{\ln(z)}$$

Find the second derivative of the following function:

$$y = \frac{4x+5}{x}$$

$$\frac{dy}{dx} = \frac{x\cdot 4-(4x+5)\cdot 1}{x^2} = -\frac{5}{x^2}$$

$$\frac{d^2y}{dx^2} = \frac{10}{x^3} > 0$$

Is the above function convex, concave or neither for all x>0? Prove.

The above function is strictly convex since the second derivative is positive for all positive values of x.