Economics 214 Name____

Quiz 6 Fall 2007

Definitions or Equations (10 pts)

Implicit function theorem:

For an implicit function $F(y, x_1, \cdots, x_n) = k,$ which is defined at the point $(y^0, x_1^0, \cdots, x_n^0)$ and has continuous first partial derivatives at that with $F_y(y^0, x_1^0, \cdots, x_n^0) \neq 0,$ there is a function $y = f(x_1, \cdots, x_n)$ defined in the neighborhood of $(x_1^0, \cdots, x_n^0)$ corresponding to $F(y, x_1, \cdots, x_n) = k$ such that

I. $F\left(f(x_1^0, \cdots, x_n^0), x_1^0, \cdots, x_n^0\right) = k$

II. $y^0 = f(x_1^0, \cdots, x_n^0)$

III. $f_i(x_1^0, \cdots, x_n^0) = -\dfrac{F_{x_i}(y^0, x_1^0, \cdots, x_n^0)}{F_y(y^0, x_1^0, \cdots, x_n^0)}$

Inflection Point:

The twice-differentiable function *f(x)* has an inflection point at $\tilde{x}$ if and only if the sign of the second derivative switches from one sign prior to the point to the opposite sign after the point.

Homogeneous Function:

A function $y = f(x_1, \cdots x_n)$ is **homogeneous of degree k** if, for an number *s* where *s*>0,

$s^k y = f(sx_1, \cdots, sx_n)$.

Stationary Point:

We say that *x** is a stationary point of a differentiable function *f(x)* if *f'(x*)*=0.

Second Order condition for an local extremeum:

If the second derivate of the differentiable function y=f(x) is negative when evaluated at a stationary point x* (that is, *f"(x*)*<0) , then that stationary point represents a local minimum of that function. If the second derivative at the stationary point is positive (that is, *f"(x*)*>0), then that stationary point represents a local maximum of that function.

Problems (10 pts)

The implicit function $\overline{U}=U(A, B)$ shows the what combinations of apples (*A*) and bananas (*B*) provide the level of utility $\overline{U}$. The absolute value of the slope of the indifference curve is the marginal rate of substitution (MRS), which measures the rate at which one good can be substituted for another while maintaining the same level of utility. Find the derivative of the implicit function to determine the MRS of apples for bananas (MRS_{AB}) for the utility function $\overline{U}=\sqrt{AB}$.

$$\frac{dB}{dA}=-\frac{\partial F}{\partial A}/\frac{\partial F}{\partial B}=-(\frac{1}{2}A^{-1/2}B^{1/2})/(\frac{1}{2}A^{1/2}B^{-1/2})=-\frac{B}{A}$$

Therefore, MRS_{AB} = B/A.

Consider a monopolist's linear demand function *P=12-2Q*, where *P* is the price of the good and *Q* is its quantity. The monopolist's total cost function is $TC=\frac{1}{3}Q^3-5Q^2+17Q+25$. Determine the level of output at which the monopolist should produce in order to maximize her profits. What is the optimal price that corresponds with this quantity?

$$TR=PQ=12Q-2Q^2$$

$$\Pi=TR-TC=(12Q-2Q^2)-(\frac{1}{3}Q^3-5Q^2+17Q+25)=-\frac{1}{3}Q^3+3Q^2-5Q-25$$

$$\frac{d\Pi}{dQ}=-Q^2+6Q-5=(-Q+1)(Q-5)=0$$

The stationary points are Q* = 1 and Q** = 5.

$$\frac{d^2\Pi}{dQ^2}=-2Q+6$$

The second derivative is positive at X* and negative at X**, therefore the profit maximizing quantity is Q** = 5. At Q**=5, P**=2.