Chapter 15 Homework

Problem 15.1

a) \[ P = \begin{bmatrix} (1/\sigma_1)I_{T_1} & 0 \\ 0 & (1/\sigma_2)I_{T_2} \end{bmatrix} \]

\[ \hat{\beta} = (X'W^{-1}X)^{-1}X'W^{-1}y \]

b) \[
\begin{bmatrix}
X' & X_2
\end{bmatrix}
\begin{bmatrix}
\frac{1}{\sigma_1^2}I_{13} & 0 \\
0 & \frac{1}{\sigma_2^2}I_{13}
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2
\end{bmatrix}
\begin{bmatrix}
\frac{1}{\sigma_1^2}I_{13} & 0 \\
0 & \frac{1}{\sigma_2^2}I_{13}
\end{bmatrix}
\begin{bmatrix}
1 \\
y
\end{bmatrix}
\]

\[ = \begin{bmatrix}
\frac{1}{\sigma_1^2}X_1' + \frac{1}{\sigma_2^2}X_2' \\
\frac{1}{\sigma_1^2}X_1 \\
\frac{1}{\sigma_2^2}X_2
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2
\end{bmatrix}
\begin{bmatrix}
\frac{1}{\sigma_1^2}X_1' + \frac{1}{\sigma_2^2}X_2' \\
\frac{1}{\sigma_1^2}X_1 \\
\frac{1}{\sigma_2^2}X_2
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix}
\]

\[ cov(\hat{\beta}) = (X'W^{-1}X)^{-1} \]

\[ = \begin{bmatrix}
X_1' & X_2
\end{bmatrix}
\begin{bmatrix}
\frac{1}{\sigma_1^2}I_{13} & 0 \\
0 & \frac{1}{\sigma_2^2}I_{13}
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2
\end{bmatrix}
\begin{bmatrix}
\frac{1}{\sigma_1^2}I_{13} & 0 \\
0 & \frac{1}{\sigma_2^2}I_{13}
\end{bmatrix}
\begin{bmatrix}
X_1' \\
X_2'
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2
\end{bmatrix}
\]

\[ = \begin{bmatrix}
\frac{1}{\sigma_1^2}X_1' + \frac{1}{\sigma_2^2}X_2' \\
\frac{1}{\sigma_1^2}X_1 \\
\frac{1}{\sigma_2^2}X_2
\end{bmatrix}
\begin{bmatrix}
X_1' \\
X_2'
\end{bmatrix}
\begin{bmatrix}
\frac{1}{\sigma_1^2}X_1' + \frac{1}{\sigma_2^2}X_2'
\end{bmatrix}
\]

Problem 15.2

<table>
<thead>
<tr>
<th>sample 1 40</th>
</tr>
</thead>
<tbody>
<tr>
<td>read expend income</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>sample 1 15</td>
</tr>
<tr>
<td>?ols expend income</td>
</tr>
<tr>
<td>gen1 sse1 = $sse</td>
</tr>
<tr>
<td>.NOTE..CURRENT VALUE OF $SSE = 189.05</td>
</tr>
<tr>
<td>sample 26 40</td>
</tr>
<tr>
<td>?ols expend income</td>
</tr>
<tr>
<td>gen1 sse2 = $sse</td>
</tr>
<tr>
<td>.NOTE..CURRENT VALUE OF $SSE = 1268.7</td>
</tr>
<tr>
<td>* Constructing Goldfeld-Quandt test</td>
</tr>
<tr>
<td>gen1 GQ = sse2/sse1</td>
</tr>
</tbody>
</table>
The F increased from 3.349 to 6.71 when we left 10 observations out. The critical F with 13 and 13 degrees freedom is 2.69. There appears to be heteroscedasticity. The increase in the F clearly increased the probability of detecting that the null hypothesis is false when it is false.

Problem 15.

(a)

```
_ genr w1 = 1/income
_ ols expend income / weight = w1
```

REQUIRED MEMORY IS PAR= 3 CURRENT PAR= 781
OLS ESTIMATION
40 OBSERVATIONS DEPENDENT VARIABLE= EXPEND
...NOTE..SAMPLE RANGE SET TO: 1, 40
SUM OF LOG(SQRT(ABS(WEIGHT))) = -1.0012

R-SQUARE = 0.4177 R-SQUARE ADJUSTED = 0.4024
VARIANCE OF THE ESTIMATE-SIGMA**2 = 37.695
STANDARD ERROR OF THE ESTIMATE-SIGMA = 6.1396
SUM OF SQUARED ERRORS-SSE= 1432.4
MEAN OF DEPENDENT VARIABLE = 22.012
LOG OF THE LIKELIHOOD FUNCTION = -129.323

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>ESTIMATED COEFFICIENT</th>
<th>STANDARD ERROR</th>
<th>T-RATIO</th>
<th>P-VALUE</th>
<th>PARTIAL STANDARDIZED CORR. COEFFICIENT</th>
<th>STANDARDIZED ELASTICITY AT MEANS</th>
</tr>
</thead>
<tbody>
<tr>
<td>INCOME</td>
<td>0.25519</td>
<td>0.4888E-01</td>
<td>5.221</td>
<td>0.000</td>
<td>0.646</td>
<td>0.6463</td>
</tr>
<tr>
<td>CONSTANT</td>
<td>5.7821</td>
<td>3.257</td>
<td>1.776</td>
<td>0.084</td>
<td>0.0000</td>
<td>0.7373</td>
</tr>
</tbody>
</table>

| end
| STOP

(b)

```
let \( \alpha = \ln \sigma^2 \)

(i) \[ \sigma_t^2 = \exp(\alpha + y \ln x_t) = \exp(\alpha)(\exp(\ln x_t))^y = \exp(\ln \sigma^2)(\exp(\ln x_t))^y = \sigma^2 x_t^y \]

(ii) |_ sample 1 40
|_ read expend income
2 VARIABLES AND 40 OBSERVATIONS STARTING AT OBS 1

|_ ols expend income / resid = e
```

REQUIRED MEMORY IS PAR= 3 CURRENT PAR= 781
OLS ESTIMATION
40 OBSERVATIONS    DEPENDENT VARIABLE= EXPEND
...NOTE..SAMPLE RANGE SET TO:    1,    40

R-SQUARE = 0.3171     R-SQUARE ADJUSTED = 0.2991
VARIANCE OF THE ESTIMATE-SIGMA**2 = 46.853
STANDARD ERROR OF THE ESTIMATE-SIGMA = 6.8449
SUM OF SQUARED ERRORS-SSE= 1780.4
MEAN OF DEPENDENT VARIABLE = 23.595
LOG OF THE LIKELIHOOD FUNCTION = -132.672

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>ESTIMATED COEFFICIENT</th>
<th>STANDARD ERROR</th>
<th>T-RATIO</th>
<th>P-VALUE</th>
<th>CORR. COEFFICIENT</th>
<th>AT MEANS</th>
</tr>
</thead>
<tbody>
<tr>
<td>INCOME</td>
<td>0.23225</td>
<td>0.5529E-01</td>
<td>4.200</td>
<td>0.000</td>
<td>0.563</td>
<td>0.5631</td>
</tr>
<tr>
<td>CONSTANT</td>
<td>7.3832</td>
<td>4.008</td>
<td>1.842</td>
<td>0.073</td>
<td>0.286</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

(iii)

| genr esq = e*e
| genr lnesq = log(esq)
| genr lnincome = log(income)
| ols lnesq lnincome / predict = lnsighat2

REQUIRED MEMORY IS PAR=       4 CURRENT PAR=     781
OLS ESTIMATION
...NOTE..SAMPLE RANGE SET TO:    1,    40

R-SQUARE = 0.1201     R-SQUARE ADJUSTED = 0.0970
VARIANCE OF THE ESTIMATE-SIGMA**2 = 3.0357
STANDARD ERROR OF THE ESTIMATE-SIGMA = 1.7423
SUM OF SQUARED ERRORS-SSE= 115.36
MEAN OF DEPENDENT VARIABLE = 2.5604
LOG OF THE LIKELIHOOD FUNCTION = -77.9404

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>ESTIMATED COEFFICIENT</th>
<th>STANDARD ERROR</th>
<th>T-RATIO</th>
<th>P-VALUE</th>
<th>CORR. COEFFICIENT</th>
<th>AT MEANS</th>
</tr>
</thead>
<tbody>
<tr>
<td>LNINCOME</td>
<td>2.0650</td>
<td>0.9065</td>
<td>2.278</td>
<td>0.028</td>
<td>0.347</td>
<td>3.3894</td>
</tr>
<tr>
<td>CONSTANT</td>
<td>-6.1179</td>
<td>3.820</td>
<td>-1.602</td>
<td>0.118</td>
<td>-0.251</td>
<td>-2.3894</td>
</tr>
</tbody>
</table>

| confid lnincome
USING 95% AND 90% CONFIDENCE INTERVALS

CONFIDENCE INTERVALS BASED ON T-DISTRIBUTION WITH 38 D.F.
- T CRITICAL VALUES = 2.021 AND 1.684

<table>
<thead>
<tr>
<th>NAME</th>
<th>LOWER 2.5%</th>
<th>LOWER 5%</th>
<th>COEFFICIENT</th>
<th>UPPER 5%</th>
<th>UPPER 2.5%</th>
<th>STD. ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>LNINCOME</td>
<td>0.2328</td>
<td>0.5383</td>
<td>2.0650</td>
<td>3.592</td>
<td>3.897</td>
<td>0.907</td>
</tr>
</tbody>
</table>

Neither null hypothesis would be rejected at the 5% level of significance since both 1 and 2 are within the 95% confidence interval.
### (iv)

```plaintext
|_ genr sighat2 = exp(lnsighat2)
|_ print sighat2

SIGHAT2
|       1.815569  |  3.262973  |  5.076779  |  6.181047  |  6.608874 |
|  6.745242    |  6.998984  |  7.681964  |  8.492790  |  8.603720 |
| 15.06941     | 15.07805   | 15.17768   | 15.17768   | 15.70740  |
| 20.00231     | 20.38944   | 20.42483   | 24.90670   | 24.90670  |
```

### (v)

```plaintext
|_ genr w = sqrt(sighat2)
|_ ols expend income / weight = w

REQUIRED MEMORY IS PAR=       5 CURRENT PAR=     781

OLS ESTIMATION
|  40 OBSERVATIONS  | DEPENDENT VARIABLE= EXPEND
| ...NOTE..SAMPLE RANGE SET TO:      1,     40
| SUM OF LOG(SQRT(ABS(WEIGHT)))  = -0.91469

R-SQUARE =   0.2523     R-SQUARE ADJUSTED =   0.2326
VARIANCE OF THE ESTIMATE-SIGMA**2 =    56.999
STANDARD ERROR OF THE ESTIMATE-SIGMA =   7.5498
SUM OF SQUARED ERRORS-SSE=   2166.0
MEAN OF DEPENDENT VARIABLE =   24.907
LOG OF THE LIKELIHOOD FUNCTION = -137.507

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>ESTIMATED NAME</th>
<th>STANDARD ERROR</th>
<th>T-RATIO</th>
<th>PARTIAL STANDARDIZED</th>
<th>ELASTICITY AT MEANS</th>
</tr>
</thead>
<tbody>
<tr>
<td>INCOME</td>
<td>0.22033</td>
<td>0.6153E-01</td>
<td>3.581</td>
<td>0.001</td>
<td>0.6675</td>
</tr>
<tr>
<td>CONSTANT</td>
<td>8.2816</td>
<td>4.794</td>
<td>1.727</td>
<td>0.092</td>
<td>0.3325</td>
</tr>
</tbody>
</table>

|_ end
|_ STOP
```

The slope coefficient and its standard error did not change that much, there was slightly greater changes in the estimate of the constant and its standard error.

Problem 15.6

(a)

```plaintext
|_ sample 1 28
|_ read c1 q1 c2 q2
|  4 VARIABLES AND   28 OBSERVATIONS STARTING AT OBS     1

|_ genr w = 1/sqrt(q1)
|_ genr q12 = q1*q1
|_ genr q13 = q12*q1

|_ ols c1 q1 q12 q13 / weight=w

REQUIRED MEMORY IS PAR=       4 CURRENT PAR=     781
```
OLS ESTIMATION
28 OBSERVATIONS DEPENDENT VARIABLE= C1
...NOTE..SAMPLE RANGE SET TO: 1, 28
SUM OF LOG(SQRT(ABS(WEIGHT))) = -0.55168

R-SQUARE = 0.9863 R-SQUARE ADJUSTED = 0.9846
VARIANCE OF THE ESTIMATE-SIGMA**2 = 280.03
STANDARD ERROR OF THE ESTIMATE-SIGMA = 16.734
SUM OF SQUARED ERRORS-SSE= 6720.8
MEAN OF DEPENDENT VARIABLE = 311.62
LOG OF THE LIKELIHOOD FUNCTION = -117.013

<table>
<thead>
<tr>
<th>VARIABLE</th>
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<th>P-VALUE</th>
<th>CORR. COEFFICIENT</th>
<th>AT MEANS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>74.803</td>
<td>20.14</td>
<td>3.715</td>
<td>0.001</td>
<td>0.604</td>
<td>1.4235</td>
</tr>
<tr>
<td>Q12</td>
<td>-12.042</td>
<td>4.113</td>
<td>-2.928</td>
<td>0.007</td>
<td>-0.513</td>
<td>-2.4749</td>
</tr>
<tr>
<td>Q13</td>
<td>1.0880</td>
<td>0.2534</td>
<td>4.294</td>
<td>0.000</td>
<td>0.659</td>
<td>2.0875</td>
</tr>
<tr>
<td>CONSTANT</td>
<td>85.335</td>
<td>28.71</td>
<td>2.973</td>
<td>0.007</td>
<td>0.519</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

(b)
|_ test
|_ test constant = 0
|_ test q13 = 0
|_ end

F STATISTIC = 92.628317 WITH 2 AND 24 D.F. P-VALUE= 0.00000
WALD CHI-SQUARE STATISTIC = 185.25663 WITH 2 D.F. P-VALUE= 0.00000
UPPER BOUND ON P-VALUE BY CHEBYCHEV INEQUALITY = 0.01080

We reject the null hypothesis.

(c) If null hypothesis is true then the average cost function is linear. In our case everywhere downward sloping (AC = 74.803 – 12.042Q)

Problem 15.8

| sample 1 26
| read q p t
3 VARIABLES AND 26 OBSERVATIONS STARTING AT OBS 1

| genr x = DUM(t-13)
| ?ols q p t / resid = e
| genr esq = e*e
| ols esq x

REQUIRED MEMORY IS PAR= 3 CURRENT PAR= 781
OLS ESTIMATION
26 OBSERVATIONS DEPENDENT VARIABLE= ESQ
...NOTE..SAMPLE RANGE SET TO: 1, 26

R-SQUARE = 0.2974 R-SQUARE ADJUSTED = 0.2681
VARIANCE OF THE ESTIMATE-SIGMA**2 = 0.22537E+06
STANDARD ERROR OF THE ESTIMATE-SIGMA = 474.73
SUM OF SQUARED ERRORS-SSE= 0.54090E+07
MEAN OF DEPENDENT VARIABLE = 352.67
LOG OF THE LIKELIHOOD FUNCTION = -196.084

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>ESTIMATED COEFFICIENT</th>
<th>STANDARD ERROR</th>
<th>T-RATIO</th>
<th>24 DF</th>
<th>P-VALUE</th>
<th>PARTIAL STANDARDIZED ELASTICITY CORR. COEFFICIENT AT MEANS</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>-593.42</td>
<td>186.2</td>
<td>-3.187</td>
<td>0.004</td>
<td>-0.545</td>
<td>-0.5453</td>
</tr>
<tr>
<td>CONSTANT</td>
<td>649.39</td>
<td>131.7</td>
<td>4.932</td>
<td>0.000</td>
<td>0.709</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

_. gen1 ssr=$ssr
..NOTE..CURRENT VALUE OF $SSR = 0.22890E+07
_. gen1 sig2=$sig2
..NOTE..CURRENT VALUE OF $SIG2= 0.22537E+06
_. gen1 BP = ssr/(2*(sig2**2))
_. print BP
BP
 0.2253252E-04
_. end
_. STOP

Since the BP is so small we fail to reject the null hypothesis of no heteroscedasticity at any meaningful level.