Chapter 15 Homework

Problem 15.1

a) $P=\begin{bmatrix}(1/\sigma_1)I_{T_1} & 0\\ 0 & (1/\sigma_2)I_{T_2}\end{bmatrix}$

$$\hat{\beta}=\left(X'W^{-1}X\right)^{-1}X'W^{-1}y$$

b) $$\left[\begin{pmatrix}X'_1 & X_2\end{pmatrix}\begin{bmatrix}\frac{1}{\sigma_1^2}I_{13} & 0\\ 0 & \frac{1}{\sigma_2^2}I_{13}\end{bmatrix}\begin{pmatrix}X_1\\ X_2\end{pmatrix}\right]^{-1}\begin{pmatrix}X'_1 & X'_2\end{pmatrix}\begin{bmatrix}\frac{1}{\sigma_1^2}I_{13} & 0\\ 0 & \frac{1}{\sigma_2^2}I_{13}\end{bmatrix}\begin{pmatrix}y_1\\ y_2\end{pmatrix}$$

$$=\left[\begin{bmatrix}\frac{1}{\sigma_1^2}X'_1 & \frac{1}{\sigma_2^2}X'_2\end{bmatrix}\begin{pmatrix}X_1\\ X_2\end{pmatrix}\right]^{-1}\begin{bmatrix}\frac{1}{\sigma_1^2}X'_1 & \frac{1}{\sigma_2^2}X'_2\end{bmatrix}\begin{pmatrix}y_1\\ y_2\end{pmatrix}$$

$$=\left[\frac{1}{\sigma_1^2}X'_1X_1+\frac{1}{\sigma_2^2}X'_2X_2\right]^{-1}\left[\frac{1}{\sigma_1^2}X'_1y_1+\frac{1}{\sigma_2^2}X'_2y_2\right]$$

$$cov(\hat{\beta})=(X'W^{-1}X)^{-1}$$

$$=\left[\begin{pmatrix}X'_1 & X_2\end{pmatrix}\begin{bmatrix}\frac{1}{\sigma_1^2}I_{13} & 0\\ 0 & \frac{1}{\sigma_2^2}I_{13}\end{bmatrix}\begin{pmatrix}X_1\\ X_2\end{pmatrix}\right]^{-1}$$

$$=\left[\begin{bmatrix}\frac{1}{\sigma_1^2}X'_1 & \frac{1}{\sigma_2^2}X'_2\end{bmatrix}\begin{pmatrix}X_1\\ X_2\end{pmatrix}\right]^{-1}=\left[\frac{1}{\sigma_1^2}X'_1X_1+\frac{1}{\sigma_2^2}X'_2X_2\right]^{-1}$$

Problem 15.2

```
|_ sample 1 40
 |_ read expend income
    2 VARIABLES AND       40 OBSERVATIONS STARTING AT OBS       1

 |_ sample 1 15
 |_ ?ols expend income
 |_ gen1 sse1 = $sse
 ..NOTE..CURRENT VALUE OF $SSE =   189.05
 |_ sample 26 40
 |_ ?ols expend income
 |_ gen1 sse2 = $sse
 ..NOTE..CURRENT VALUE OF $SSE =   1268.7
 |_ * Constructing Goldfeld-Quandt test
 |_ gen1 GQ = sse2/sse1
```

```
|_ print GQ
GQ
   6.710954
|_ end
|_ end
|_ STOP
```

The F increased from 3.349 to 6.71 when we left 10 observations out. The critical F with 13 and 13 degrees freedom is 2.69. There appears to be heteroscedasticity. The increase in the F clearly increased the probability of detecting that the null hypothesis is false when it is false.

Problem 15.

(a)

```
|_ genr w1 = 1/income

|_ ols expend income / weight = w1

REQUIRED MEMORY IS PAR=       3 CURRENT PAR=     781
 OLS ESTIMATION
       40 OBSERVATIONS     DEPENDENT VARIABLE= EXPEND
...NOTE..SAMPLE RANGE SET TO:      1,     40
SUM OF LOG(SQRT(ABS(WEIGHT)))  =  -1.0012

 R-SQUARE =   0.4177     R-SQUARE ADJUSTED =   0.4024
VARIANCE OF THE ESTIMATE-SIGMA**2 =   37.695
STANDARD ERROR OF THE ESTIMATE-SIGMA =   6.1396
SUM OF SQUARED ERRORS-SSE=   1432.4
MEAN OF DEPENDENT VARIABLE =   22.012
LOG OF THE LIKELIHOOD FUNCTION = -129.323


VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      38 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
INCOME    0.25519     0.4888E-01   5.221     0.000 0.646     0.6463     0.7373
CONSTANT   5.7821      3.257       1.776     0.084 0.277     0.0000     0.2627
|_ end
|_ STOP
```

(b)

$$let \;\; \alpha = \ln \sigma^2$$

(i) $\sigma_t^2 = \exp(\alpha + \gamma \ln x_t) = \exp(\alpha)(\exp(\ln x_t))^\gamma = \exp(\ln \sigma^2)(\exp(\ln x_t))^\gamma$

$$= \sigma^2 x_t^\gamma$$

```
  (ii) |_ sample 1 40
|_ read expend income
   2 VARIABLES AND       40 OBSERVATIONS STARTING AT OBS       1


|_ ols expend income / resid = e

REQUIRED MEMORY IS PAR=       3 CURRENT PAR=     781
 OLS ESTIMATION
```

```
       40 OBSERVATIONS     DEPENDENT VARIABLE= EXPEND
 ...NOTE..SAMPLE RANGE SET TO:      1,     40

  R-SQUARE =   0.3171     R-SQUARE ADJUSTED =   0.2991
 VARIANCE OF THE ESTIMATE-SIGMA**2 =   46.853
 STANDARD ERROR OF THE ESTIMATE-SIGMA =   6.8449
 SUM OF SQUARED ERRORS-SSE=   1780.4
 MEAN OF DEPENDENT VARIABLE =   23.595
 LOG OF THE LIKELIHOOD FUNCTION = -132.672


 VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
   NAME    COEFFICIENT   ERROR      38 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
 INCOME    0.23225     0.5529E-01   4.200     0.000 0.563     0.5631     0.6871
 CONSTANT   7.3832      4.008       1.842     0.073 0.286     0.0000     0.3129

 (iii)

|_ genr esq = e*e
 |_ genr lnesq = log(esq)
 |_ genr lnincome = log(income)

 |_ ols lnesq lnincome / predict = lnsighat2

 REQUIRED MEMORY IS PAR=       4 CURRENT PAR=     781
  OLS ESTIMATION
       40 OBSERVATIONS     DEPENDENT VARIABLE= LNESQ
 ...NOTE..SAMPLE RANGE SET TO:      1,     40

  R-SQUARE =   0.1201     R-SQUARE ADJUSTED =   0.0970
 VARIANCE OF THE ESTIMATE-SIGMA**2 =   3.0357
 STANDARD ERROR OF THE ESTIMATE-SIGMA =   1.7423
 SUM OF SQUARED ERRORS-SSE=   115.36
 MEAN OF DEPENDENT VARIABLE =   2.5604
 LOG OF THE LIKELIHOOD FUNCTION = -77.9404


 VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
   NAME    COEFFICIENT   ERROR      38 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
 LNINCOME   2.0650     0.9065       2.278     0.028 0.347     0.3466     3.3894
 CONSTANT  -6.1179      3.820      -1.602     0.118-0.251     0.0000    -2.3894

  |_ confid lnincome
 USING 95% AND 90% CONFIDENCE INTERVALS

 CONFIDENCE INTERVALS BASED ON T-DISTRIBUTION WITH  38 D.F.
      - T CRITICAL VALUES =   2.021 AND   1.684
 NAME   LOWER 2.5%   LOWER 5%   COEFFICIENT   UPPER 5%   UPPER 2.5%   STD. ERROR
 LNINCOME  0.2328      0.5383      2.0650       3.592       3.897         0.907
```

Neither null hypothesis would be rejected at the 5% level of significance since both 1 and 2 are within the 95% confidence interval.

(iv)

```
|_ genr sighat2= exp(lnsighat2)
|_ print sighat2
 SIGHAT2
    1.815569       3.262973       5.076779       6.181047       6.608874
    6.745242       6.998984       7.681964       8.429790       8.603720
    9.126491       9.935270       10.04017       10.60926       10.75043
    11.48199       12.58259       12.75653       14.40278       14.42814
    15.06941       15.07805       15.17768       15.17768       15.70740
    16.06724       16.30046       17.01054       19.23951       19.64873
    20.00231       20.38944       20.42483       24.90670       24.90670
    25.55522       26.82565       30.57839       39.03681       39.98201
```

(v)

```
|_ genr w = sqrt(sighat2)
 |_ ols expend income / weight = w

 REQUIRED MEMORY IS PAR=       5 CURRENT PAR=     781
  OLS ESTIMATION
        40 OBSERVATIONS     DEPENDENT VARIABLE= EXPEND
 ...NOTE..SAMPLE RANGE SET TO:      1,     40
 SUM OF LOG(SQRT(ABS(WEIGHT)))  = -0.91469

  R-SQUARE =   0.2523     R-SQUARE ADJUSTED =   0.2326
 VARIANCE OF THE ESTIMATE-SIGMA**2 =   56.999
 STANDARD ERROR OF THE ESTIMATE-SIGMA =   7.5498
 SUM OF SQUARED ERRORS-SSE=   2166.0
 MEAN OF DEPENDENT VARIABLE =   24.907
 LOG OF THE LIKELIHOOD FUNCTION = -137.507


 VARIABLE   ESTIMATED  STANDARD   T-RATIO         PARTIAL STANDARDIZED ELASTICITY
   NAME    COEFFICIENT   ERROR      38 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
 INCOME    0.22033     0.6153E-01   3.581     0.001 0.502     0.5023     0.6675
 CONSTANT   8.2816      4.794       1.727     0.092 0.270     0.0000     0.3325
 |_ end
 |_ STOP
```

The slope coefficient and its standard error did not change that much, there was slightly greater changes in the estimate of the constant and its standard error.

Problem 15.6

(a)

```
 |_ sample 1 28
 |_ read c1 q1 c2 q2
    4 VARIABLES AND       28 OBSERVATIONS STARTING AT OBS       1

 |_ genr w = 1/sqrt(q1)
 |_ genr q12 = q1*q1
 |_ genr q13 = q12*q1

 |_ ols c1 q1 q12 q13 / weight=w

 REQUIRED MEMORY IS PAR=       4 CURRENT PAR=     781
```

```
 OLS ESTIMATION
       28 OBSERVATIONS     DEPENDENT VARIABLE= C1
...NOTE..SAMPLE RANGE SET TO:      1,     28
SUM OF LOG(SQRT(ABS(WEIGHT)))  = -0.55168

 R-SQUARE =   0.9863     R-SQUARE ADJUSTED =   0.9846
VARIANCE OF THE ESTIMATE-SIGMA**2 =   280.03
STANDARD ERROR OF THE ESTIMATE-SIGMA =   16.734
SUM OF SQUARED ERRORS-SSE=   6720.8
MEAN OF DEPENDENT VARIABLE =   311.62
LOG OF THE LIKELIHOOD FUNCTION = -117.013


VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      24 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
Q1         74.803      20.14       3.715     0.001 0.604     1.4235     1.1196
Q12       -12.042      4.113      -2.928     0.007-0.513    -2.4749    -1.0854
Q13        1.0880     0.2534       4.294     0.000 0.659     2.0875     0.6919
CONSTANT   85.335      28.71       2.973     0.007 0.519     0.0000     0.2738
```

```
(b)
|_ test
 |_ test constant = 0
 |_ test q13 = 0
 |_ end
 F STATISTIC =   92.628317     WITH    2 AND   24 D.F.  P-VALUE= 0.00000
 WALD CHI-SQUARE STATISTIC =   185.25663     WITH    2 D.F.  P-VALUE= 0.00000
 UPPER BOUND ON P-VALUE BY CHEBYCHEV INEQUALITY = 0.01080
```

We reject the null hypothesis.

(c) If null hypothesis is true then the average cost function is linear. In our case everywhere downward sloping (AC = 74.803 – 12.042Q)

Problem 15.8

```
 |_ sample 1 26
 |_ read q p t
    3 VARIABLES AND       26 OBSERVATIONS STARTING AT OBS       1

 |_ genr x = DUM(t-13)
 |_ ?ols q p t / resid = e
 |_ genr esq = e*e

 |_ ols esq x

 REQUIRED MEMORY IS PAR=       3 CURRENT PAR=     781
  OLS ESTIMATION
        26 OBSERVATIONS     DEPENDENT VARIABLE= ESQ
 ...NOTE..SAMPLE RANGE SET TO:      1,     26

  R-SQUARE =   0.2974     R-SQUARE ADJUSTED =   0.2681
 VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.22537E+06
 STANDARD ERROR OF THE ESTIMATE-SIGMA =   474.73
 SUM OF SQUARED ERRORS-SSE=  0.54090E+07
 MEAN OF DEPENDENT VARIABLE =   352.67
```

```
LOG OF THE LIKELIHOOD FUNCTION = -196.084


VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      24 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
X         -593.42      186.2      -3.187     0.004-0.545    -0.5453    -0.8413
CONSTANT   649.39      131.7       4.932     0.000 0.709     0.0000     1.8413
|_ gen1 ssr=$ssr
..NOTE..CURRENT VALUE OF $SSR =  0.22890E+07
|_ gen1 sig2=$sig2
..NOTE..CURRENT VALUE OF $SIG2=  0.22537E+06
|_ gen1 BP = ssr/(2*(sig2**2))
|_ print BP
BP
  0.2253252E-04
|_ end
|_ STOP
```

Since the BP is so small we fail to reject the null hypothesis of no heteroscedasticity at any meaningful level.