Chapter 16
Homework

Problem 16.1

a) $var(e_t)=\rho^2 var(e_{t-1})+var(\epsilon_t)$ or $\sigma_e^2=\rho^2\sigma_e^2+\sigma_\epsilon^2$ *Hence* $\sigma_e^2=\frac{1}{(1-\rho^2)}\sigma_\epsilon^2$

By repeat substitution, we can write e_t as

$$e_t=\sum_{i=0}^{k-1}\rho^i\epsilon_{t-i}+\rho^k e_{t-k}$$

$$cov(e_t e_{t-k})=E[e_t e_{t-k}]=E[e_{t-k}\sum_{i=0}^{k-1}\rho^i\epsilon_{t-i}]+\rho^k E[e_{t-k}^2]=0+\frac{\rho^k}{(1-\rho^2)}\sigma_\epsilon^2=\frac{\rho^k}{(1-\rho^2)}\sigma_\epsilon^2$$

Using the above, we can now demonstrate

$$E[ee']=E\begin{bmatrix} e_t^2 & e_t e_{t-1} & e_t e_{t-2} & e_t e_{t-3} \\ e_{t-1}e_t & e_{t-1}^2 & e_{t-1}e_{t-2} & e_{t-1}e_{t-3} \\ e_{t-2}e_t & e_{t-2}e_{t-1} & e_{t-2}^2 & e_{t-2}e_{t-3} \\ e_{t-3}e_t & e_{t-3}e_{t-1} & e_{t-3}e_{t-2} & e_{t-3}^2 \end{bmatrix} Using\,the\,above, we\,get$$

$$=\frac{\sigma_\epsilon^2}{(1-\rho^2)}\begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 \\ \rho & 1 & \rho & \rho^2 \\ \rho^2 & \rho & 1 & \rho \\ \rho^3 & \rho^2 & \rho & 1 \end{bmatrix}=\sigma_\epsilon^2 V.$$

$$VV^{-1}=\frac{1}{(1-\rho^2)}\begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 \\ \rho & 1 & \rho & \rho^2 \\ \rho^2 & \rho & 1 & \rho \\ \rho^3 & \rho^2 & \rho & 1 \end{bmatrix}\begin{bmatrix} 1 & -\rho & 0 & 0 \\ -\rho & 1+\rho^2 & -\rho & 0 \\ 0 & -\rho & 1+\rho^2 & -\rho \\ 0 & 0 & -\rho & 1 \end{bmatrix}$$

b) $$=\frac{1}{(1-\rho^2)}\begin{bmatrix} 1-\rho^2 & -\rho+\rho+\rho^3-\rho^3 & -\rho^2+\rho^2+\rho^4-\rho^4 & -\rho^3+\rho^3 \\ \rho-\rho & -\rho^2+1+\rho^2-\rho^2 & -\rho+\rho+\rho^3-\rho^3 & -\rho^2+\rho^2 \\ \rho^2-\rho^2 & -\rho^3+\rho+\rho^3-\rho & -\rho^2+1+\rho^2-\rho^2 & -\rho+\rho \\ \rho^3-\rho^3 & -\rho^4+\rho^2+\rho^4-\rho^2 & -\rho^3+\rho+\rho^3-\rho & -\rho^2+1 \end{bmatrix}$$

$$=\frac{1}{(1-\rho^2)}\begin{bmatrix} 1-\rho^2 & 0 & 0 & 0 \\ 0 & 1-\rho^2 & 0 & 0 \\ 0 & 0 & 1-\rho^2 & 0 \\ 0 & 0 & 0 & 1-\rho^2 \end{bmatrix}=I$$

c)

$$P'P=\begin{bmatrix}\sqrt{1-\rho^2} & -\rho & 0 & 0\\ 0 & 1 & -\rho & 0\\ 0 & 0 & 1 & -\rho\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\sqrt{1-\rho^2} & 0 & 0 & 0\\ -\rho & 1 & 0 & 0\\ 0 & -\rho & 1 & 0\\ 0 & 0 & -\rho & 1\end{bmatrix}=\begin{bmatrix}1-\rho^2+\rho^2 & -\rho & 0 & 0\\ -\rho & 1+\rho^2 & -\rho & 0\\ 0 & -\rho & 1+\rho^2 & -\rho\\ 0 & 0 & -\rho & 1\end{bmatrix}$$

$$=\begin{bmatrix}1 & -\rho & 0 & 0\\ -\rho & 1+\rho^2 & -\rho & 0\\ 0 & -\rho & 1+\rho^2 & -\rho\\ 0 & 0 & -\rho & 1\end{bmatrix}=V^{-1}$$

d) $$y^*=Py=\begin{bmatrix}\sqrt{1-\rho^2} & 0 & 0 & 0\\ -\rho & 1 & 0 & 0\\ 0 & -\rho & 1 & 0\\ 0 & 0 & -\rho & 1\end{bmatrix}\begin{bmatrix}y_1\\ y_2\\ y_3\\ y_4\end{bmatrix}=\begin{bmatrix}\sqrt{1-\rho^2}\,y_1\\ y_2-\rho y_1\\ y_3-\rho y_2\\ y_4-\rho y_3\end{bmatrix}$$

Problem 16.2

a) R-SQUARE = 0.8163 R-SQUARE ADJUSTED = 0.8027

```
VARIANCE OF THE ESTIMATE-SIGMA**2 =   11.087
STANDARD ERROR OF THE ESTIMATE-SIGMA =   3.3296
SUM OF SQUARED ERRORS-SSE=   299.34
MEAN OF DEPENDENT VARIABLE =   20.222
LOG OF THE LIKELIHOOD FUNCTION = -77.0737
```

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 27 DF	P-VALUE	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
Y	0.76991	0.7179E-01	10.72	0.000	0.900	0.8869	0.8201
R	-0.18420	0.1264	-1.457	0.157	-0.270	-0.1205	-0.1279
CONSTANT	6.2249	2.511	2.479	0.020	0.431	0.0000	0.3078

As expected investment is positively related to income and negatively (although not significant) related to the interest rate.

b) DURBIN-WATSON STATISTIC = 0.85215

```
DURBIN-WATSON POSITIVE AUTOCORRELATION TEST P-VALUE =    0.000114
              NEGATIVE AUTOCORRELATION TEST P-VALUE =    0.999886
```

Given the p-value we reject the null hypothesis of no autocorrelation in favor of the alternative hypothesis of autocorrelation.

c) LEAST SQUARES ESTIMATION 30 OBSERVATIONS

```
BY COCHRANE-ORCUTT TYPE PROCEDURE WITH CONVERGENCE = 0.00100
```

ITERATION	RHO	LOG L.F.	SSE
1	0.00000	-77.0737	299.34
2	0.56773	-70.4731	190.29
3	0.62336	-70.4272	189.06
4	0.62720	-70.4295	189.04

```
      5            0.62749       -70.4297            189.04

 LOG L.F. =   -70.4297       AT RHO =     0.62749

                  ASYMPTOTIC  ASYMPTOTIC  ASYMPTOTIC
          ESTIMATE    VARIANCE    ST.ERROR     T-RATIO
RHO        0.62749     0.02021     0.14216     4.41413

 R-SQUARE =   0.8840     R-SQUARE ADJUSTED =   0.8754
VARIANCE OF THE ESTIMATE-SIGMA**2 =   7.0014
STANDARD ERROR OF THE ESTIMATE-SIGMA =   2.6460
SUM OF SQUARED ERRORS-SSE=   189.04
MEAN OF DEPENDENT VARIABLE =   20.222
LOG OF THE LIKELIHOOD FUNCTION = -70.4297


VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      27 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
Y         0.73378     0.1254       5.853     0.000 0.748     0.8453     0.7816
R        -0.28938     0.7661E-01  -3.777     0.001-0.588    -0.1893    -0.2010
CONSTANT   8.7043      3.111       2.798     0.009 0.474     0.0000     0.4304
```

The income coefficient is practically the same as the OLS estimate. The interest rate coefficient is more negative and implies slightly greater degree of interest rate elasticity. The original regression suggested that the interest rate was insignificant, but after correcting for autocorrelation the coefficient of interest rate is significant.

d) |_ FC / BLUP beg=31 end=31 list

```
REQUIRED MEMORY IS PAR=       2 CURRENT PAR=     781
..ASSUMING ESTIMATION ENDED AT OBSERVATION    31
DEPENDENT VARIABLE = I               1 OBSERVATIONS
REGRESSION COEFFICIENTS
  0.733778950814     -0.289377323469       8.70425673843
AUTOCORRELATION RHO
  0.6274948745769
USER SPECIFIED RHO= 0.62749
USER SPECIFIED SRHO= 0.00000
..FORECAST STD. ERRORS USE JUDGE(1985, EQ. 8.3.13)
IGNORE FORECASTS AND STD. ERRORS BEFORE OBS.   32

    OBS.   OBSERVED     PREDICTED   CALCULATED  STD. ERROR
     NO.    VALUE        VALUE       RESIDUAL
     31   30.000       31.069      -1.0690       3.150          *    I

BLUP IS ACTIVE - PREDICTED VALUES ADJUSTED WITH LAGGED RESIDUALS
SUM OF ABSOLUTE ERRORS=   1.0690
R-SQUARE BETWEEN OBSERVED AND PREDICTED = 0.0000
MEAN ERROR =  -1.0690
SUM-SQUARED ERRORS =   1.1428
MEAN SQUARE ERROR =   1.1428
MEAN ABSOLUTE ERROR=   1.0690
ROOT MEAN SQUARE ERROR =   1.0690
MEAN SQUARED PERCENTAGE ERROR=   12.698
THEIL INEQUALITY COEFFICIENT U = 0.000
```

```
   DECOMPOSITION
      PROPORTION DUE TO BIAS =   1.0000
      PROPORTION DUE TO VARIANCE =  0.45515E-13
      PROPORTION DUE TO COVARIANCE = -0.45515E-13
   DECOMPOSITION
      PROPORTION DUE TO BIAS =   1.0000
      PROPORTION DUE TO REGRESSION =  0.45515E-13
      PROPORTION DUE TO DISTURBANCE = -0.45515E-13
```

Problem 16.3

```
a) R-SQUARE =   0.8299     R-SQUARE ADJUSTED =   0.8222

 VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.81221E-01
 STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.28499
 SUM OF SQUARED ERRORS-SSE=   1.7869
 MEAN OF DEPENDENT VARIABLE =  0.63427
 LOG OF THE LIKELIHOOD FUNCTION = -2.88344


 VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
   NAME    COEFFICIENT   ERROR      22 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
 LNU       -1.6116     0.1555      -10.36     0.000-0.911    -0.9110    -4.5224
 CONSTANT   3.5027     0.2829       12.38     0.000 0.935     0.0000     5.5224

 |_ confid lnU
 USING 95% AND 90% CONFIDENCE INTERVALS

 CONFIDENCE INTERVALS BASED ON T-DISTRIBUTION WITH  22 D.F.
      - T CRITICAL VALUES =   2.074 AND   1.717
 NAME   LOWER 2.5%   LOWER 5%   COEFFICIENT   UPPER 5%   UPPER 2.5%   STD. ERROR
 LNU       -1.934      -1.879      -1.6116      -1.345      -1.289         0.156

b) DURBIN-WATSON STATISTIC  =   1.08962

 DURBIN-WATSON POSITIVE AUTOCORRELATION TEST P-VALUE =    0.008821
               NEGATIVE AUTOCORRELATION TEST P-VALUE =    0.991179
```

The original assumption is wrong. Given the DW we reject the null hypothesis of no autocorrelation. The errors are not uncorrelated, but autocorrelated. Since the standdard error estimate for the coefficient of lnU is biased, the confidence interval is biased.

```
c) LEAST SQUARES ESTIMATION             24 OBSERVATIONS

 BY COCHRANE-ORCUTT TYPE PROCEDURE WITH CONVERGENCE = 0.00100

     ITERATION          RHO              LOG L.F.           SSE
         1            0.00000        -2.88344             1.7869
         2            0.44719       -0.381923             1.4372
         3            0.44708       -0.381881             1.4372

   LOG L.F. =  -0.381881       AT RHO =     0.44708

                      ASYMPTOTIC  ASYMPTOTIC  ASYMPTOTIC
           ESTIMATE    VARIANCE    ST.ERROR     T-RATIO
```

```
 RHO        0.44708     0.03334     0.18259     2.44857

  R-SQUARE =   0.8632     R-SQUARE ADJUSTED =   0.8570
 VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.65328E-01
 STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.25559
 SUM OF SQUARED ERRORS-SSE=   1.4372
 MEAN OF DEPENDENT VARIABLE =  0.63427
 LOG OF THE LIKELIHOOD FUNCTION =-0.381881


 VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
   NAME    COEFFICIENT   ERROR      22 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
 LNU       -1.6160     0.1232      -13.11     0.000-0.942    -0.9135    -4.5348
 CONSTANT   3.5137     0.2377       14.78     0.000 0.953     0.0000     5.5397

 |_ confid LnU
 USING 95% AND 90% CONFIDENCE INTERVALS

 CONFIDENCE INTERVALS BASED ON T-DISTRIBUTION WITH  22 D.F.
      - T CRITICAL VALUES =   2.074 AND   1.717
 NAME   LOWER 2.5%   LOWER 5%   COEFFICIENT   UPPER 5%   UPPER 2.5%   STD. ERROR
 LNU       -1.872      -1.828      -1.6160      -1.404      -1.360         0.123
```

The new estimated confidence interval is narrower since the ols estimate of the standard error was upward bias given positive autocorrelation.

Problem 16.4

a) For Profit maximizing we need MR=MC.

$$MR = \beta_1 + 2\beta_2 Q$$
$$MC = \alpha_2 + 2\alpha_3 Q$$

When MR=MC The profit maximizing Q is

$$Q^* = \frac{(\beta_1 - \alpha_2)}{[2(\alpha_3 - \beta_2)]}$$

b) |_ *Estimating Total Revenue Function with OLS

```
 |_ ols tr q qsq / noconstant dw

 REQUIRED MEMORY IS PAR=      22 CURRENT PAR=     781
  OLS ESTIMATION
       48 OBSERVATIONS     DEPENDENT VARIABLE= TR
 ...NOTE..SAMPLE RANGE SET TO:      1,     48

  R-SQUARE =   0.6166     R-SQUARE ADJUSTED =   0.6083
 VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.13905E+07
 STANDARD ERROR OF THE ESTIMATE-SIGMA =   1179.2
 SUM OF SQUARED ERRORS-SSE=  0.63962E+08
 MEAN OF DEPENDENT VARIABLE =   13847.
 LOG OF THE LIKELIHOOD FUNCTION = -406.571
 RAW MOMENT R-SQUARE =   0.9932
```

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 46 DF	P-VALUE	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
Q	174.28	4.540	38.39	0.000	0.985	4.5207	2.0301
QSQ	-0.50243	0.2354E-01	-21.34	0.000	-0.953	-4.2640	-1.0288

```
|_ *Estimating Total Cost Function with OLS


|_ ols tc q qsq / dw

REQUIRED MEMORY IS PAR=      23 CURRENT PAR=     781
 OLS ESTIMATION
       48 OBSERVATIONS     DEPENDENT VARIABLE= TC
...NOTE..SAMPLE RANGE SET TO:      1,     48

 R-SQUARE =   0.9862     R-SQUARE ADJUSTED =   0.9856
VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.19411E+06
STANDARD ERROR OF THE ESTIMATE-SIGMA =   440.58
SUM OF SQUARED ERRORS-SSE=  0.87351E+07
MEAN OF DEPENDENT VARIABLE =   8211.3
LOG OF THE LIKELIHOOD FUNCTION = -358.789
```

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 45 DF	P-VALUE	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
Q	13.400	9.005	1.488	0.144	0.217	0.1786	0.2632
QSQ	0.18717	0.2753E-01	6.800	0.000	0.712	0.8161	0.6463
CONSTANT	742.92	692.8	1.072	0.289	0.158	0.0000	0.0905

$$Q^{*}=\frac{(\beta_1-\alpha_2)}{[2(\alpha_3-\beta_2)]}$$

$$Q^{*}=\frac{(174.28-13.4)}{(2(0.18717-(-0.50243)))}=116.65 \ \ \mathit{units}$$

c) Testing autocorrelation in Total Revenue Function

```
DURBIN-WATSON STATISTIC  =   0.28671
DURBIN-WATSON POSITIVE AUTOCORRELATION TEST P-VALUE =    0.000000
              NEGATIVE AUTOCORRELATION TEST P-VALUE =    1.000000
```

Testing for autocorrelation in the Total Cost Function

```
DURBIN-WATSON STATISTIC  =   0.47899
DURBIN-WATSON POSITIVE AUTOCORRELATION TEST P-VALUE =    0.000000
              NEGATIVE AUTOCORRELATION TEST P-VALUE =    1.000000
```

Both functions have significant Durbin-Watson statistic and hence are plagued with autocorrelation.

d) DEPENDENT VARIABLE = TR
..NOTE..R-SQUARE,ANOVA,RESIDUALS DONE ON ORIGINAL VARS

LEAST SQUARES ESTIMATION 48 OBSERVATIONS
BY COCHRANE-ORCUTT TYPE PROCEDURE WITH CONVERGENCE = 0.00100

ITERATION	RHO	LOG L.F.	SSE
1	0.00000	-406.571	0.63962E+08
2	0.88817	-369.655	0.13299E+08
3	0.94743	-369.497	0.13014E+08
4	0.94992	-369.519	0.13014E+08
5	0.94999	-369.520	0.13013E+08

LOG L.F. = -369.520 AT RHO = 0.94999

	ESTIMATE	ASYMPTOTIC VARIANCE	ASYMPTOTIC ST.ERROR	ASYMPTOTIC T-RATIO
RHO	0.94999	0.00203	0.04507	21.07602

R-SQUARE = 0.9220 R-SQUARE ADJUSTED = 0.9203
VARIANCE OF THE ESTIMATE-SIGMA**2 = 0.28290E+06
STANDARD ERROR OF THE ESTIMATE-SIGMA = 531.89
SUM OF SQUARED ERRORS-SSE= 0.13013E+08
MEAN OF DEPENDENT VARIABLE = 13847.
LOG OF THE LIKELIHOOD FUNCTION = -369.520
RAW MOMENT R-SQUARE = 0.9986

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 46 DF	P-VALUE	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
Q	171.26	7.181	23.85	0.000	0.962	4.4424	1.9949
QSQ	-0.50754	0.2233E-01	-22.73	0.000	-0.958	-4.3073	-1.0393

DEPENDENT VARIABLE = TC
..NOTE..R-SQUARE,ANOVA,RESIDUALS DONE ON ORIGINAL VARS

LEAST SQUARES ESTIMATION 48 OBSERVATIONS
BY COCHRANE-ORCUTT TYPE PROCEDURE WITH CONVERGENCE = 0.00100

ITERATION	RHO	LOG L.F.	SSE
1	0.00000	-358.789	0.87351E+07
2	0.78607	-338.546	0.36834E+07
3	0.80983	-338.529	0.36728E+07
4	0.81005	-338.529	0.36727E+07

LOG L.F. = -338.529 AT RHO = 0.81005

	ESTIMATE	ASYMPTOTIC VARIANCE	ASYMPTOTIC ST.ERROR	ASYMPTOTIC T-RATIO
RHO	0.81005	0.00716	0.08463	9.57111

R-SQUARE = 0.9942 R-SQUARE ADJUSTED = 0.9939
VARIANCE OF THE ESTIMATE-SIGMA**2 = 81616.
STANDARD ERROR OF THE ESTIMATE-SIGMA = 285.69
SUM OF SQUARED ERRORS-SSE= 0.36727E+07
MEAN OF DEPENDENT VARIABLE = 8211.3
LOG OF THE LIKELIHOOD FUNCTION = -338.529

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 45 DF	P-VALUE	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
Q	6.2790	4.618	1.360	0.181	0.199	0.0837	0.1233
QSQ	0.20930	0.1425E-01	14.69	0.000	0.910	0.9125	0.7227
CONSTANT	1209.3	399.2	3.030	0.004	0.412	0.0000	0.1473

e)

$$Q^{*}=\frac{(\beta_1-\alpha_2)}{[2(\alpha_3-\beta_2)]}$$

$$Q^{*}=\frac{(171.29-6.279)}{(2(0.20931-(-0.50754)))}=115.09 \quad units$$

f) Total Revenue = 171.26*115.09 - 0.50754 *$(115.09)^2$ = $12,987.59
Total Cost = 1209.3 + 6.279*115.09 + 0.2093*$(115.09)^2$ = $4,704.28
Total Profit = TR – TC = $12,987.59 - $4,704.28 = $8,283.31