Chapter 17
Problem Solutions

Problem 17.1

a)

$$E(\underline{e})=E\begin{pmatrix}\underline{e}_1\\ \underline{e}_2\end{pmatrix}=\begin{pmatrix}E(\underline{e}_1)\\ E(\underline{e}_2)\end{pmatrix}=\begin{pmatrix}\underline{0}\\ \underline{0}\end{pmatrix}$$

$$E(\underline{e}\underline{e}')=E\left(\begin{pmatrix}\underline{e}_1\\ \underline{e}_2\end{pmatrix}\begin{pmatrix}\underline{e}_1\\ \underline{e}_2\end{pmatrix}'\right)=E\begin{pmatrix}\underline{e}_1\underline{e}_1' & \underline{e}_1\underline{e}_2'\\ \underline{e}_2\underline{e}_1' & \underline{e}_2\underline{e}_2'\end{pmatrix}=\begin{pmatrix}\sigma_1^2 I_T & 0\\ 0 & \sigma_2^2 I_T\end{pmatrix}=W$$

$$\underline{\hat{\beta}}=(X'W^{-1}X)^{-1}X'W^{-1}\underline{y}=\left(\begin{pmatrix}X_1' & 0\\ 0 & X_2'\end{pmatrix}\begin{pmatrix}\frac{1}{\sigma_1^2}I_T & 0\\ 0 & \frac{1}{\sigma_2^2}I_T\end{pmatrix}\begin{pmatrix}X_1 & 0\\ 0 & X_2\end{pmatrix}\right)^{-1}\begin{pmatrix}X_1' & 0\\ 0 & X_2'\end{pmatrix}\begin{pmatrix}\frac{1}{\sigma_1^2}I_T & 0\\ 0 & \frac{1}{\sigma_2^2}I_T\end{pmatrix}\begin{pmatrix}\underline{y}_1\\ \underline{y}_2\end{pmatrix}=$$

$$\begin{pmatrix}\frac{1}{\sigma_1^2}X_1'X_1 & 0\\ 0 & \frac{1}{\sigma_2^2}X_2'X_2\end{pmatrix}^{-1}\begin{pmatrix}\frac{1}{\sigma_1^2}X_1'\underline{y}_1\\ \frac{1}{\sigma_2^2}X_2'\underline{y}_2\end{pmatrix}=\begin{pmatrix}\sigma_1^2(X_1'X_1)^{-1} & 0\\ 0 & \sigma_2^2(X_2'X_2)^{-1}\end{pmatrix}\begin{pmatrix}\frac{1}{\sigma_1^2}X_1'\underline{y}_1\\ \frac{1}{\sigma_2^2}X_2'\underline{y}_2\end{pmatrix}=$$

$$\begin{pmatrix}(X_1'X_1)^{-1}X_1'\underline{y}_1\\ (X_2'X_2)^{-1}X_2'\underline{y}_2\end{pmatrix}=\begin{pmatrix}\underline{b}_1\\ \underline{b}_2\end{pmatrix}\ i.e.\ \text{the OLS estimators.}$$

b)

$$\hat{\beta}=(X'W^{-1}X)^{-1}X'W^{-1}y=(X'W^{-1}X)^{-1}X'W^{-1}(X'\beta+e)=$$
$$(X'W^{-1}X)^{-1}X'W^{-1}X\beta+(X'W^{-1}X)^{-1}X'W^{-1}e=$$
$$\beta+(X'W^{-1}X)^{-1}X'W^{-1}e$$
$$E[\hat{\beta}]=E[\beta]+(X'W^{-1}X)^{-1}X'W^{-1}E[e]=\beta$$
$$since E[e]=0.$$

c) Since by the Gauss-Markov theorem the generalized least squares estimator is the best linear unbiased estimator, I would demonstrate that the least-squares estimator is also another linear unbiased estimator. Without appealing to the Gauss-Markov theorem, one would demonstrate that the variance-covariance matrix of the least squares estimator $(X'X)^{-1}(X'WX)(X'X)^{-1}$ exceeded the variance-covariance matrix of the generalized least squares estimator, $(X'W^{-1}X)^{-1}$, by a positive semi-definite matrix.

We will use $\langle x \rangle$ to *denote the Knonecker product symbol.*

$$When\, X_1 = X_2 = Z, X = I\langle x\rangle z,\ \ W = \Sigma\langle x\rangle I,\ where\, \Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix},$$

$$W^{-1} = \Sigma^{-1}\langle x\rangle I.\ \ now$$

d) $\hat{\beta} = (X'W^{-1}X)^{-1}X'W^{-1}y = \left((I\langle x\rangle Z')(\Sigma^{-1}\langle x\rangle I)(I\langle x\rangle Z)\right)^{-1}(I\langle x\rangle Z')(\Sigma^{-1}\langle x\rangle I)y =$

$$(\Sigma^{-1}\langle x\rangle Z'Z)^{-1}(\Sigma^{-1}\langle x\rangle Z')y = (\Sigma\langle x\rangle (Z'Z)^{-1})(\Sigma^{-1}\langle x\rangle Z')y = (I\langle x\rangle (Z'Z)^{-1}Z')y =$$

$$\begin{bmatrix} (Z'Z)^{-1}Z' & 0 \\ 0 & (Z'Z)^{-1}Z' \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} (Z'Z)^{-1}Z'y_1 \\ (Z'Z)^{-1}Z'y_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

the OLS estimator.

Problem 17.4

a) |_ ols lnq1 lnp1 lny / pcov

```
REQUIRED MEMORY IS PAR=       6 CURRENT PAR=     781
 OLS ESTIMATION
       30 OBSERVATIONS     DEPENDENT VARIABLE= LNQ1
...NOTE..SAMPLE RANGE SET TO:      1,     30

 R-SQUARE =   0.6234     R-SQUARE ADJUSTED =   0.5955
VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.15531
STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.39410
SUM OF SQUARED ERRORS-SSE=   4.1935
MEAN OF DEPENDENT VARIABLE =   2.2123
LOG OF THE LIKELIHOOD FUNCTION = -13.0532


VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      27 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
LNP1     -0.56627     0.2148      -2.636     0.014-0.452    -0.3118    -0.5297
LNY        1.4337     0.2287       6.270     0.000 0.770     0.7415     3.8668
CONSTANT  -5.1704      1.415      -3.654     0.001-0.575     0.0000    -2.3371

VARIANCE-COVARIANCE MATRIX OF COEFFICIENTS
LNP1      0.46137E-01
LNY      -0.25052E-02  0.52288E-01
CONSTANT -0.80530E-01 -0.30680       2.0025
             LNP1         LNY          CONSTANT

|_ ols lnq2 lnp2 lny / pcov

REQUIRED MEMORY IS PAR=       6 CURRENT PAR=     781
 OLS ESTIMATION
       30 OBSERVATIONS     DEPENDENT VARIABLE= LNQ2
...NOTE..SAMPLE RANGE SET TO:      1,     30

 R-SQUARE =   0.5412     R-SQUARE ADJUSTED =   0.5072
VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.20307
STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.45063
SUM OF SQUARED ERRORS-SSE=   5.4828
MEAN OF DEPENDENT VARIABLE =   1.9486
```

```
 LOG OF THE LIKELIHOOD FUNCTION = -17.0744


 VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
   NAME    COEFFICIENT   ERROR      27 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
 LNP2     -0.64794     0.1875      -3.455     0.002-0.554    -0.4506    -0.6329
 LNY        1.1439     0.2612       4.379     0.000 0.644     0.5710     3.5027
 CONSTANT  -3.6434      1.609      -2.264     0.032-0.399     0.0000    -1.8698

 VARIANCE-COVARIANCE MATRIX OF COEFFICIENTS
 LNP2      0.35162E-01
 LNY       0.11527E-02  0.68224E-01
 CONSTANT -0.73801E-01 -0.40927       2.5892
              LNP2         LNY          CONSTANT

 |_ ols lnq3 lnp3 lny / pcov

 REQUIRED MEMORY IS PAR=       6 CURRENT PAR=     781
  OLS ESTIMATION
       30 OBSERVATIONS     DEPENDENT VARIABLE= LNQ3
 ...NOTE..SAMPLE RANGE SET TO:      1,     30

  R-SQUARE =   0.9147     R-SQUARE ADJUSTED =   0.9084
 VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.34930E-01
 STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.18689
 SUM OF SQUARED ERRORS-SSE=  0.94310
 MEAN OF DEPENDENT VARIABLE =   3.5992
 LOG OF THE LIKELIHOOD FUNCTION =  9.32859


 VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
   NAME    COEFFICIENT   ERROR      27 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
 LNP3     -0.96408     0.6533E-01  -14.76     0.000-0.943    -0.8297    -0.5200
 LNY       0.87090     0.1083       8.038     0.000 0.840     0.4519     1.4438
 CONSTANT  0.27425     0.6633      0.4134     0.683 0.079     0.0000     0.0762

 VARIANCE-COVARIANCE MATRIX OF COEFFICIENTS
 LNP3      0.42676E-02
 LNY       0.20839E-03  0.11739E-01
 CONSTANT -0.95276E-02 -0.70448E-01  0.44000
              LNP3         LNY          CONSTANT
```

b) ITERATION 0 COEFFICIENTS

```
  -0.56627       1.4337     -0.64794       1.1439     -0.96408      0.87090

 ITERATION    0 SIGMA
   0.13978
  -0.19135E-01  0.18276
  -0.40329E-01 -0.37133E-01  0.31437E-01
```

c) BREUSCH-PAGAN LM TEST FOR DIAGONAL COVARIANCE MATRIX = 18.733

```
 CHI-SQUARE WITH    3 D.F.     P-VALUE= 0.00031

 LOG OF DETERMINANT OF SIGMA=  -8.3171
 LOG OF LIKELIHOOD FUNCTION =  -2.94766
```

Reject Ho. Matrix is not diagonal.

```
EQUATION  1 OF  3 EQUATIONS
 DEPENDENT VARIABLE = LNQ1                 30 OBSERVATIONS

  R-SQUARE =   0.5878
 VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.16999
 STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.41230
 SUM OF SQUARED ERRORS-SSE=   4.5898
 MEAN OF DEPENDENT VARIABLE =   2.2123
 LOG OF THE LIKELIHOOD FUNCTION =  5.11127


 VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
   NAME    COEFFICIENT   ERROR      27 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
 LNP1     -0.90937     0.1374      -6.617     0.000-0.786    -0.5007    -0.8507
 LNY        1.4523     0.2285       6.356     0.000 0.774     0.7511     3.9171
 CONSTANT  -4.5715      1.386      -3.299     0.003-0.536     0.0000    -2.0664

 VARIANCE-COVARIANCE MATRIX OF COEFFICIENTS
 LNP1      0.18887E-01
 LNY      -0.10255E-02  0.52208E-01
 CONSTANT -0.32967E-01 -0.30939       1.9199
             LNP1         LNY          CONSTANT
  EQUATION  2 OF  3 EQUATIONS
 DEPENDENT VARIABLE = LNQ2                 30 OBSERVATIONS

  R-SQUARE =   0.5184
 VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.21314
 STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.46168
 SUM OF SQUARED ERRORS-SSE=   5.7549
 MEAN OF DEPENDENT VARIABLE =   1.9486
 LOG OF THE LIKELIHOOD FUNCTION =  5.11127


 VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
   NAME    COEFFICIENT   ERROR      27 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
 LNP2     -0.86500     0.1319      -6.559     0.000-0.784    -0.6015    -0.8449
 LNY        1.1368     0.2612       4.353     0.000 0.642     0.5675     3.4809
 CONSTANT  -3.1878      1.585      -2.012     0.054-0.361     0.0000    -1.6360

 VARIANCE-COVARIANCE MATRIX OF COEFFICIENTS
 LNP2      0.17391E-01
 LNY       0.57014E-03  0.68205E-01
 CONSTANT -0.36502E-01 -0.40805       2.5113
             LNP2         LNY          CONSTANT
  EQUATION  3 OF  3 EQUATIONS
 DEPENDENT VARIABLE = LNQ3                 30 OBSERVATIONS

  R-SQUARE =   0.9138
 VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.35297E-01
 STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.18788
 SUM OF SQUARED ERRORS-SSE=  0.95303
 MEAN OF DEPENDENT VARIABLE =   3.5992
 LOG OF THE LIKELIHOOD FUNCTION =  5.11127
```

```
 VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
   NAME    COEFFICIENT   ERROR      27 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
 LNP3     -0.99891     0.3647E-01  -27.39     0.000-0.982    -0.8597    -0.5387
 LNY       0.86919     0.1083       8.025     0.000 0.839     0.4510     1.4409
 CONSTANT  0.35201     0.6522      0.5397     0.594 0.103     0.0000     0.0978

 VARIANCE-COVARIANCE MATRIX OF COEFFICIENTS
 LNP3      0.13297E-02
 LNY       0.64929E-04  0.11732E-01
 CONSTANT -0.29686E-02 -0.70127E-01  0.42537
              LNP3         LNY          CONSTANT
```

e) Equation 1

OLS: VARIANCE-COVARIANCE MATRIX OF COEFFICIENTS

LNP1	0.46137E-01		
LNY	-0.25052E-02	0.52288E-01	
CONSTANT	-0.80530E-01	-0.30680	2.0025
	LNP1	LNY	CONSTANT

SUR: VARIANCE-COVARIANCE MATRIX OF COEFFICIENTS

LNP1	0.18887E-01		
LNY	-0.10255E-02	0.52208E-01	
CONSTANT	-0.32967E-01	-0.30939	1.9199
	LNP1	LNY	CONSTANT

The SUR variances are slightly smaller.

Equation 2

OLS: VARIANCE-COVARIANCE MATRIX OF COEFFICIENTS

LNP2	0.35162E-01		
LNY	0.11527E-02	0.68224E-01	
CONSTANT	-0.73801E-01	-0.40927	2.5892
	LNP2	LNY	CONSTANT

SUR: VARIANCE-COVARIANCE MATRIX OF COEFFICIENTS

LNP2	0.17391E-01		
LNY	0.57014E-03	0.68205E-01	
CONSTANT	-0.36502E-01	-0.40805	2.5113
	LNP2	LNY	CONSTANT

The SUR variances are slightly smaller.

Equation 3

OLS: VARIANCE-COVARIANCE MATRIX OF COEFFICIENTS

LNP3	0.42676E-02		
LNY	0.20839E-03	0.11739E-01	
CONSTANT	-0.95276E-02	-0.70448E-01	0.44000
	LNP3	LNY	CONSTANT

SUR: VARIANCE-COVARIANCE MATRIX OF COEFFICIENTS

LNP3	0.13297E-02		
LNY	0.64929E-04	0.11732E-01	
CONSTANT	-0.29686E-02	-0.70127E-01	0.42537
	LNP3	LNY	CONSTANT

The SUR variances are slightly smaller.

f) They are elasticities of price and income respectively.

g) |_ confid lnp1 lnp2 lnp3

```
USING 95% AND 90% CONFIDENCE INTERVALS

CONFIDENCE INTERVALS BASED ON T-DISTRIBUTION WITH  27 D.F.
     - T CRITICAL VALUES =   2.052 AND   1.703
NAME   LOWER 2.5%   LOWER 5%   COEFFICIENT   UPPER 5%   UPPER 2.5%   STD. ERROR
LNP1      -1.191      -1.143     -0.90937     -0.6753     -0.6274        0.137
LNP2      -1.136      -1.090     -0.86500     -0.6404     -0.5944        0.132
LNP3      -1.074      -1.061     -0.99891     -0.9368     -0.9241        0.036
```

For all 3 goods the price elasticity can range from elastic to inelastic.

h) EQUATION 1 OF 3 EQUATIONS

```
DEPENDENT VARIABLE = LNQ1                 30 OBSERVATIONS

 R-SQUARE =   0.5783
VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.16193
STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.40241
SUM OF SQUARED ERRORS-SSE=   4.6961
MEAN OF DEPENDENT VARIABLE =   2.2123
LOG OF THE LIKELIHOOD FUNCTION =  3.71627


VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      29 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
LNP1     -0.95265     0.4511E-01  -21.12     0.000-0.969    -0.5245    -0.8911
LNY        1.4547     0.2298       6.331     0.000 0.762     0.7523     3.9234
CONSTANT  -4.4959      1.375      -3.269     0.003-0.519     0.0000    -2.0323
 EQUATION  2 OF  3 EQUATIONS
DEPENDENT VARIABLE = LNQ2                 30 OBSERVATIONS

 R-SQUARE =   0.4963
VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.20755
STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.45558
SUM OF SQUARED ERRORS-SSE=   6.0190
MEAN OF DEPENDENT VARIABLE =   1.9486
LOG OF THE LIKELIHOOD FUNCTION =  3.71627


VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      29 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
LNP2     -0.95265     0.4511E-01  -21.12     0.000-0.969    -0.6624    -0.9305
LNY        1.1339     0.2604       4.355     0.000 0.629     0.5660     3.4721
CONSTANT  -3.0039      1.559      -1.927     0.064-0.337     0.0000    -1.5416
 EQUATION  3 OF  3 EQUATIONS
DEPENDENT VARIABLE = LNQ3                 30 OBSERVATIONS

 R-SQUARE =   0.9146
VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.32557E-01
STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.18044
SUM OF SQUARED ERRORS-SSE=  0.94417
MEAN OF DEPENDENT VARIABLE =   3.5992
```

```
 LOG OF THE LIKELIHOOD FUNCTION =  3.71627


 VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
   NAME    COEFFICIENT   ERROR      29 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
 LNP3     -0.95265     0.4511E-01  -21.12     0.000-0.969    -0.8199    -0.5138
 LNY       0.87145     0.1215      7.172     0.000 0.800     0.4522     1.4447
 CONSTANT  0.24874     0.7327     0.3395     0.737 0.063     0.0000     0.0691
```

The common estimator is more elastic than coefficient of log price for equation 1 and 2 from both the ols and sur estimates. The common coefficient is less elastic than the estimate for equation 3 for ols and sur.

i) |_ test

```
 |_ test lnp1=lnp2
 |_ test lnp1=lnp3
 |_ end
 F STATISTIC =  0.56906409     WITH    2 AND   81 D.F.  P-VALUE= 0.56830
 WALD CHI-SQUARE STATISTIC =   1.1381282     WITH    2 D.F.  P-VALUE= 0.56605
 UPPER BOUND ON P-VALUE BY CHEBYCHEV INEQUALITY = 1.00000
```

We cannot reject the null hypothesis that the 3 equations have a common price elasticity.

Problem 17.5

a)|_ ols lnq1 lnp1 lnp2 lnp3 lny

```
 REQUIRED MEMORY IS PAR=       6 CURRENT PAR=     781
  OLS ESTIMATION
        30 OBSERVATIONS     DEPENDENT VARIABLE= LNQ1
 ...NOTE..SAMPLE RANGE SET TO:      1,     30

  R-SQUARE =   0.6545     R-SQUARE ADJUSTED =   0.5992
 VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.15388
 STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.39228
 SUM OF SQUARED ERRORS-SSE=   3.8471
 MEAN OF DEPENDENT VARIABLE =   2.2123
 LOG OF THE LIKELIHOOD FUNCTION = -11.7599


 VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
   NAME    COEFFICIENT   ERROR      25 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
 LNP1     -0.57294     0.2200      -2.604     0.015-0.462    -0.3154    -0.5360
 LNP2      0.72491E-01 0.1683      0.4306     0.670 0.086     0.0522     0.0624
 LNP3     -0.20332     0.1382      -1.471     0.154-0.282    -0.1744    -0.1784
 LNY        1.4265     0.2277       6.265     0.000 0.782     0.7377     3.8474
 CONSTANT  -4.8570      1.506      -3.225     0.003-0.542     0.0000    -2.1954

 |_ ols lnq2 lnp1 lnp2 lnp3 lny

 REQUIRED MEMORY IS PAR=       6 CURRENT PAR=     781
  OLS ESTIMATION
```

```
       30 OBSERVATIONS     DEPENDENT VARIABLE= LNQ2
...NOTE..SAMPLE RANGE SET TO:      1,     30

 R-SQUARE =   0.5618     R-SQUARE ADJUSTED =   0.4917
VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.20945
STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.45766
SUM OF SQUARED ERRORS-SSE=   5.2362
MEAN OF DEPENDENT VARIABLE =   1.9486
LOG OF THE LIKELIHOOD FUNCTION = -16.3843


VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      25 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
LNP1      0.19088     0.2567      0.7435     0.464 0.147     0.1014     0.2027
LNP2     -0.63234     0.1964     -3.220      0.004-0.541    -0.4397    -0.6177
LNP3      0.13513     0.1613      0.8378     0.410 0.165     0.1119     0.1346
LNY        1.1406     0.2656       4.294     0.000 0.652     0.5694     3.4927
CONSTANT  -4.3110      1.757      -2.454     0.021-0.441     0.0000    -2.2124

|_ ols lnq3 lnp1 lnp2 lnp3 lny

REQUIRED MEMORY IS PAR=       6 CURRENT PAR=     781
 OLS ESTIMATION
       30 OBSERVATIONS     DEPENDENT VARIABLE= LNQ3
...NOTE..SAMPLE RANGE SET TO:      1,     30

 R-SQUARE =   0.9381     R-SQUARE ADJUSTED =   0.9282
VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.27372E-01
STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.16545
SUM OF SQUARED ERRORS-SSE=  0.68431
MEAN OF DEPENDENT VARIABLE =   3.5992
LOG OF THE LIKELIHOOD FUNCTION =  14.1400


VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      25 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
LNP1     -0.25039     0.9281E-01  -2.698     0.012-0.475    -0.1383    -0.1440
LNP2     -0.14429     0.7100E-01  -2.032     0.053-0.377    -0.1043    -0.0763
LNP3     -0.96517     0.5831E-01  -16.55     0.000-0.957    -0.8307    -0.5205
LNY       0.87971     0.9603E-01   9.161     0.000 0.878     0.4565     1.4584
CONSTANT   1.0166     0.6351       1.601     0.122 0.305     0.0000     0.2824

b)|_ genr zero = 0

|_ genr l=1
|_ matrix x1=l|lnp1|lnp2|lnp3|lny
|_ matrix z=zero|zero|zero|zero|zero
|_ matrix d1=x1'|z'|z'
|_ matrix d2=z'|x1'|z'
|_ matrix d3=z'|z'|x1'
|_ matrix X=d1'|d2'|d3'
|_ matrix yt=lnq1'|lnq2'|lnq3'
|_ matrix y=yt'
|_ matrix b=inv(X'*X)*X'*y
|_ print b
B
  -4.856952     -0.5729425      0.7249065E-01 -0.2033165       1.426510
```

```
   -4.311034      0.1908794     -0.6323395      0.1351330       1.140617
    1.016591     -0.2503892     -0.1442914     -0.9651710      0.8797076
 |_ sample 1 15
 |_ genr crt=0
 |_ print crt
 CRT
    0.000000       0.000000       0.000000       0.000000       0.000000
    0.000000       0.000000       0.000000       0.000000       0.000000
    0.000000       0.000000       0.000000       0.000000       0.000000
 |_ sample 1 30
 |_ matrix cr1=crt'
 |_ matrix cr2=crt'
 |_ matrix cr3=crt'
 |_ gen1 cr1:3=1
 |_ gen1 cr1:7=-1
 |_ gen1 cr2:4=1
 |_ gen1 cr2:12=-1
 |_ gen1 cr3:9=1
 |_ gen1 cr3:13=-1
 |_ matrix rt=cr1'|cr2'|cr3'
 |_ matrix r=rt'
 |_ matrix bhat = b-inv(X'X)*r'inv(r*inv(X'X)*r')*r*b
 |_ print bhat
 BHAT
   -4.954920     -0.5615122      0.1117693     -0.2052987       1.427080 (Eq 1)
   -3.919914      0.1117693     -0.6312177      0.2073811E-01   1.139363 (Eq 2)
   0.6195287     -0.2052987      0.2073811E-01 -0.9777231      0.8820565 (Eq 3)

c)BREUSCH-PAGAN LM TEST FOR DIAGONAL COVARIANCE MATRIX =   23.760

 CHI-SQUARE WITH    3 D.F.     P-VALUE= 0.00003

 LOG OF DETERMINANT OF SIGMA=  -9.4885
 LOG OF LIKELIHOOD FUNCTION =   14.6230
```

We reject the null hypothesis of a diagonal variance-covariance matrix.

```
d) |_ system 3 / restrict

 |_ ols lnq1 lnp1 lnp2 lnp3 lny
 |_ ols lnq2 lnp1 lnp2 lnp3 lny
 |_ ols lnq3 lnp1 lnp2 lnp3 lny
 |_ restrict lnp2:1=lnp1:2
 |_ restrict lnp3:1=lnp1:3
 |_ restrict lnp3:2=lnp2:3
 |_ end

 MULTIVARIATE REGRESSION--    3 EQUATIONS
   12 RIGHT-HAND SIDE VARIABLES IN SYSTEM
 MAX ITERATIONS =       1         CONVERGENCE TOLERANCE =   0.10000E-02
        30 OBSERVATIONS
 IR OPTION IN EFFECT - ITERATIVE RESTRICTIONS

 ITERATION     0 COEFFICIENTS
  -0.56151      0.11177     -0.20530       1.4271      0.11177     -0.63122
   0.20738E-01   1.1394     -0.20530      0.20738E-01 -0.97772      0.88206
```

```
ITERATION    0 SIGMA
  0.12852
 -0.12062E-01  0.17853
 -0.37902E-01 -0.32337E-01  0.27741E-01

BREUSCH-PAGAN LM TEST FOR DIAGONAL COVARIANCE MATRIX =   18.613
CHI-SQUARE WITH    3 D.F.     P-VALUE= 0.00033

LOG OF DETERMINANT OF SIGMA=  -8.4588
LOG OF LIKELIHOOD FUNCTION = -0.822037

ITERATION    1 SIGMA INVERSE
   18.427
   7.3589       10.039
   33.755       21.757       107.53

ITERATION     1 COEFFICIENTS
 -0.59140      0.36895E-01 -0.20452       1.4263      0.36895E-01 -0.69235
 -0.10188       1.1354     -0.20452     -0.10188     -0.91808      0.88091

ITERATION    1 SIGMA
  0.12848
 -0.11311E-01  0.19218
 -0.39424E-01 -0.36448E-01  0.23900E-01
LOG OF DETERMINANT OF SIGMA=  -9.3710
LOG OF LIKELIHOOD FUNCTION =   12.8601

SYSTEM R-SQUARE =  0.9980
TEST OF THE OVERALL SIGNIFICANCE =   186.05
CHI-SQUARE WITH    9 D.F.     P-VALUE= 0.00000

LIKELIHOOD RATIO TEST OF DIAGONAL COVARIANCE MATRIX =   60.342
CHI-SQUARE WITH    3 D.F.     P-VALUE= 0.00000

VARIABLE       COEFFICIENT    ST.ERROR       T-RATIO
LNP1       -0.59140        0.18585        -3.1821
LNP2        0.36895E-01    0.12943        0.28507
LNP3       -0.20452        0.75676E-01    -2.7026
LNY          1.4263        0.22351         6.3813
LNP1        0.36895E-01    0.12943        0.28507
LNP2       -0.69235        0.18791        -3.6844
LNP3       -0.10188        0.67191E-01    -1.5162
LNY          1.1354        0.26319         4.3141
LNP1       -0.20452        0.75676E-01    -2.7026
LNP2       -0.10188        0.67191E-01    -1.5162
LNP3       -0.91808        0.47225E-01    -19.441
LNY         0.88091        0.10381         8.4859
 EQUATION  1 OF  3 EQUATIONS
DEPENDENT VARIABLE = LNQ1                 30 OBSERVATIONS

 R-SQUARE =   0.6539
VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.13765
STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.37102
SUM OF SQUARED ERRORS-SSE=   3.8543
MEAN OF DEPENDENT VARIABLE =   2.2123
LOG OF THE LIKELIHOOD FUNCTION =  12.8601
```

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 28 DF	P-VALUE	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
LNP1	-0.59140	0.1859	-3.182	0.004	-0.515	-0.3256	-0.5532
LNP2	0.36895E-01	0.1294	0.2851	0.778	0.054	0.0266	0.0317
LNP3	-0.20452	0.7568E-01	-2.703	0.012	-0.455	-0.1754	-0.1795
LNY	1.4263	0.2235	6.381	0.000	0.770	0.7376	3.8468
CONSTANT	-4.7473	1.377	-3.449	0.002	-0.546	0.0000	-2.1459

EQUATION 2 OF 3 EQUATIONS

DEPENDENT VARIABLE = LNQ2 30 OBSERVATIONS

R-SQUARE = 0.5176

VARIANCE OF THE ESTIMATE-SIGMA**2 = 0.20590

STANDARD ERROR OF THE ESTIMATE-SIGMA = 0.45376

SUM OF SQUARED ERRORS-SSE= 5.7653

MEAN OF DEPENDENT VARIABLE = 1.9486

LOG OF THE LIKELIHOOD FUNCTION = 12.8601

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 28 DF	P-VALUE	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
LNP1	0.36895E-01	0.1294	0.2851	0.778	0.054	0.0196	0.0392
LNP2	-0.69235	0.1879	-3.684	0.001	-0.571	-0.4814	-0.6763
LNP3	-0.10188	0.6719E-01	-1.516	0.141	-0.275	-0.0844	-0.1015
LNY	1.1354	0.2632	4.314	0.000	0.632	0.5668	3.4769
CONSTANT	-3.3872	1.612	-2.101	0.045	-0.369	0.0000	-1.7383

EQUATION 3 OF 3 EQUATIONS

DEPENDENT VARIABLE = LNQ3 30 OBSERVATIONS

R-SQUARE = 0.9352

VARIANCE OF THE ESTIMATE-SIGMA**2 = 0.25607E-01

STANDARD ERROR OF THE ESTIMATE-SIGMA = 0.16002

SUM OF SQUARED ERRORS-SSE= 0.71700

MEAN OF DEPENDENT VARIABLE = 3.5992

LOG OF THE LIKELIHOOD FUNCTION = 12.8601

VARIABLE NAME	ESTIMATED COEFFICIENT	STANDARD ERROR	T-RATIO 28 DF	P-VALUE	PARTIAL CORR.	STANDARDIZED COEFFICIENT	ELASTICITY AT MEANS
LNP1	-0.20452	0.7568E-01	-2.703	0.012	-0.455	-0.1130	-0.1176
LNP2	-0.10188	0.6719E-01	-1.516	0.141	-0.275	-0.0736	-0.0539
LNP3	-0.91808	0.4722E-01	-19.44	0.000	-0.965	-0.7901	-0.4951
LNY	0.88091	0.1038	8.486	0.000	0.849	0.4571	1.4604
CONSTANT	0.74237	0.6424	1.156	0.258	0.213	0.0000	0.2063

e)

Conparison of 3 estimators						
	Equation 1					
Variable	OLS		Restricted OLS		Restricted SUR	
	Coefficeint	St. Error	Coefficeint	St. Error	Coefficeint	St. Error
Constant	-4.85700	1.50600	-4.95492	NA	-4.74730	1.37700
lnp1	-0.57294	0.22000	-0.56151	NA	-0.59140	0.18590
lnp2	0.07249	0.01683	0.11177	NA	0.03690	0.12940
lnp3	-0.20332	0.13820	-0.20530	NA	-0.20452	0.07568
lny	1.42650	0.22770	1.42708	NA	1.42630	0.22350
	Equation 2					
Constant	-4.31100	1.75700	-3.95492	NA	-3.38720	1.61200
lnp1	0.19088	0.25670	0.11177	NA	0.03690	0.12940
lnp2	-0.63234	0.19640	-0.63122	NA	-0.69235	0.18790
lnp3	0.13513	0.16130	0.02074	NA	-0.10188	0.06719
lny	1.14060	0.26560	1.13936	NA	1.13540	0.26320
	Equation 3					
Constant	1.01660	0.63510	0.61953	NA	0.74237	0.64240
lnp1	-0.25039	0.09281	-0.20530	NA	-0.20452	0.07568
lnp2	-0.14429	0.07100	0.02074	NA	-0.10188	0.06719
lnp3	-0.96517	0.05831	-0.97772	NA	-0.91808	0.04722
lny	0.87971	0.09603	0.88206	NA	0.88091	0.10380

In general the coefficients are comparable with several exceptions. In the most cases the restricted SUR standard errors are small than the ols standard errors as they should be in theory, but don't have to be in practice.

f) |_ test

```
 |_ test lnp2:1=lnp1:2
 |_ test lnp3:1=lnp1:3
 |_ test lnp3:2=lnp2:3
 |_ end
 F STATISTIC =  0.96608713     WITH    3 AND   75 D.F.  P-VALUE= 0.41332
 WALD CHI-SQUARE STATISTIC =   2.8982614     WITH    3 D.F.  P-VALUE= 0.40758
 UPPER BOUND ON P-VALUE BY CHEBYCHEV INEQUALITY = 1.00000
```

We cannot reject the null hypothesis of symmetry in the cross elasticities.

Problem 17.7

a)

```
|_ ols lnQL lnWP

 REQUIRED MEMORY IS PAR=       3 CURRENT PAR=     781
  OLS ESTIMATION
        20 OBSERVATIONS     DEPENDENT VARIABLE= LNQL
 ...NOTE..SAMPLE RANGE SET TO:      1,     20

  R-SQUARE =   0.8559     R-SQUARE ADJUSTED =   0.8479
 VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.11548
 STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.33982
 SUM OF SQUARED ERRORS-SSE=   2.0787
 MEAN OF DEPENDENT VARIABLE =  0.11383E-01
 LOG OF THE LIKELIHOOD FUNCTION = -5.73864
```

```
 VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
   NAME    COEFFICIENT   ERROR      18 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
 LNWP       1.0835     0.1048       10.34     0.000 0.925     0.9252    -4.1524
 CONSTANT  0.58649E-01 0.7612E-01  0.7704     0.451 0.179     0.0000     5.1524

 |_ ols lnQK lnRP

 REQUIRED MEMORY IS PAR=       3 CURRENT PAR=     781
  OLS ESTIMATION
        20 OBSERVATIONS     DEPENDENT VARIABLE= LNQK
 ...NOTE..SAMPLE RANGE SET TO:      1,     20

  R-SQUARE =   0.8764     R-SQUARE ADJUSTED =   0.8695
 VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.10458
 STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.32340
 SUM OF SQUARED ERRORS-SSE=   1.8825
 MEAN OF DEPENDENT VARIABLE = -0.86034E-01
 LOG OF THE LIKELIHOOD FUNCTION = -4.74761


 VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
   NAME    COEFFICIENT   ERROR      18 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
 LNRP      0.94515     0.8366E-01   11.30     0.000 0.936     0.9362     1.5170
 CONSTANT  0.44477E-01 0.7323E-01  0.6074     0.551 0.142     0.0000    -0.5170
```

b)

```
BREUSCH-PAGAN LM TEST FOR DIAGONAL COVARIANCE MATRIX =   16.477
 CHI-SQUARE WITH    1 D.F.     P-VALUE= 0.00005

 LOG OF DETERMINANT OF SIGMA=  -6.3635
 LOG OF LIKELIHOOD FUNCTION =   6.87702
```

We reject null hypothesis of diagonal covariance matrix.

c) EQUATION 1 OF 2 EQUATIONS

```
 DEPENDENT VARIABLE = LNQL                 20 OBSERVATIONS

  R-SQUARE =   0.8411
 VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.12734
 STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.35685
 SUM OF SQUARED ERRORS-SSE=   2.2922
 MEAN OF DEPENDENT VARIABLE =  0.11383E-01
 LOG OF THE LIKELIHOOD FUNCTION =  14.0867


 VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
   NAME    COEFFICIENT   ERROR      18 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
 LNWP      0.94100     0.5027E-01   18.72     0.000 0.975     0.8035    -3.6064
 CONSTANT  0.52434E-01 0.7982E-01  0.6569     0.520 0.153     0.0000     4.6064
  EQUATION  2 OF  2 EQUATIONS
 DEPENDENT VARIABLE = LNQK                 20 OBSERVATIONS

  R-SQUARE =   0.8747
 VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.10599
 STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.32556
 SUM OF SQUARED ERRORS-SSE=   1.9078
```

```
 MEAN OF DEPENDENT VARIABLE = -0.86034E-01
 LOG OF THE LIKELIHOOD FUNCTION =  14.0867


 VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
   NAME    COEFFICIENT   ERROR      18 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
 LNRP      0.90405     0.4014E-01   22.52     0.000 0.983     0.8955     1.4510
 CONSTANT  0.38802E-01 0.7301E-01  0.5315     0.602 0.124     0.0000    -0.4510
```

d)|_ system 2 / restrict

```
 |_ ols lnQL lnWP
 |_ ols lnQK lnRP
 |_ restrict lnWP:1=lnRP:2
 |_ end

EQUATION  1 OF  2 EQUATIONS
 DEPENDENT VARIABLE = LNQL                 20 OBSERVATIONS

  R-SQUARE =   0.8344
 VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.12580
 STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.35468
 SUM OF SQUARED ERRORS-SSE=   2.3902
 MEAN OF DEPENDENT VARIABLE =  0.11383E-01
 LOG OF THE LIKELIHOOD FUNCTION =  13.9229


 VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
   NAME    COEFFICIENT   ERROR      19 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
 LNWP      0.91140     0.3455E-01   26.38     0.000 0.987     0.7782    -3.4930
 CONSTANT  0.51142E-01 0.8039E-01  0.6362     0.532 0.144     0.0000     4.4930
  EQUATION  2 OF  2 EQUATIONS
 DEPENDENT VARIABLE = LNQK                 20 OBSERVATIONS

  R-SQUARE =   0.8753
 VARIANCE OF THE ESTIMATE-SIGMA**2 =  0.99976E-01
 STANDARD ERROR OF THE ESTIMATE-SIGMA =  0.31619
 SUM OF SQUARED ERRORS-SSE=   1.8995
 MEAN OF DEPENDENT VARIABLE = -0.86034E-01
 LOG OF THE LIKELIHOOD FUNCTION =  13.9229


 VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
   NAME    COEFFICIENT   ERROR      19 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
 LNRP      0.91140     0.3455E-01   26.38     0.000 0.987     0.9027     1.4628
 CONSTANT  0.39816E-01 0.7181E-01  0.5545     0.586 0.126     0.0000    -0.4628
```

e) Yes the standard errors get smaller as we impose more restrictions.

f) |_ test lnWP:1=1

```
 TEST VALUE = -0.88603E-01 STD. ERROR OF TEST VALUE  0.34548E-01
 T STATISTIC =  -2.5646259     WITH   37 D.F.    P-VALUE= 0.01452
 F STATISTIC =   6.5773060     WITH    1 AND   37 D.F.  P-VALUE= 0.01452
 WALD CHI-SQUARE STATISTIC =   6.5773060     WITH    1 D.F.  P-VALUE= 0.01033
 UPPER BOUND ON P-VALUE BY CHEBYCHEV INEQUALITY = 0.15204
```

We reject null hypothesis that elasticity of substitution = 1 or Cobb-Douglas.