Chapter 18
Problems Solutions

Problem 18.1

a)
$$\pi_{11}=\frac{(\beta_{21}-\beta_{11})}{(\gamma_{12}-\gamma_{22})} \quad \pi_{12}=\frac{\beta_{22}}{(\gamma_{12}-\gamma_{22})}$$
$$\pi_{21}=\frac{(\gamma_{12}\beta_{21}-\beta_{11}\gamma_{22})}{(\gamma_{12}-\gamma_{22})} \quad \pi_{22}=\frac{(\gamma_{12}\beta_{22})}{(\gamma_{12}-\gamma_{22})}$$

$$\gamma_{12}=\frac{\pi_{22}}{\pi_{12}}=\frac{(\gamma_{12}\beta_{22})/(\gamma_{12}-\gamma_{22})}{((\beta_{22})/(\gamma_{12}-\gamma_{22}))}=\gamma_{12}$$
$$\beta_{11}=\pi_{21}-\gamma_{12}\pi_{11}=\frac{(\gamma_{12}\beta_{21}-\beta_{11}\gamma_{22})}{(\gamma_{12}-\gamma_{22})}-\gamma_{12}\frac{(\beta_{21}-\beta_{11})}{(\gamma_{12}-\gamma_{22})}=\frac{(\gamma_{12}\beta_{21}-\beta_{11}\gamma_{22})}{(\gamma_{12}-\gamma_{22})}-\frac{(\gamma_{12}\beta_{21}-\gamma_{12}\beta_{11})}{(\gamma_{12}-\gamma_{22})}$$
$$=\frac{(\beta_{11}(\gamma_{12}-\gamma_{22}))}{(\gamma_{12}-\gamma_{22})}=\beta_{11}$$

b)
$$X'X=\begin{bmatrix}5 & 10\\ 10 & 24\end{bmatrix} \quad X'y_1=\begin{bmatrix}22\\ 48\end{bmatrix} \quad X'y_2=\begin{bmatrix}25\\ 47\end{bmatrix}(X'X)^{-1}=\begin{bmatrix}1.2 & -0.5\\ -0.5 & 0.25\end{bmatrix}$$
$$\pi_1=(X'X)^{-1}X'y_1=\begin{bmatrix}1.2 & -0.5\\ -0.5 & 0.25\end{bmatrix}\begin{bmatrix}22\\ 48\end{bmatrix}=\begin{bmatrix}2.4\\ 1\end{bmatrix}$$
$$\pi_2=(X'X)^{-1}X'y_2=\begin{bmatrix}1.2 & -0.5\\ -0.5 & 1\end{bmatrix}\begin{bmatrix}25\\ 47\end{bmatrix}=\begin{bmatrix}6.5\\ -0.75\end{bmatrix}$$

c) Yes. Equilibrium price should increase with an increase in the wage rate and equilibrium quantity should fall with an increase in the wage rate.

d)
$$\hat{\gamma}_{12}=\hat{\pi}_{22}/\hat{\pi}_{12}=-0.75/1=-0.75$$
$$\hat{\beta}_{11}=\hat{\pi}_{21}-\hat{\gamma}_{12}\hat{\pi}_{11}=6.5-(-0.75)(2.4)=6.5-1.8=8.3$$

e) These estimates cannot be obtained as the supply equation is under identified.

Problem 18.2

(b) |_ ols p w

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REQUIRED MEMORY IS PAR=       1 CURRENT PAR=     781
 OLS ESTIMATION
       5 OBSERVATIONS     DEPENDENT VARIABLE= P
...NOTE..SAMPLE RANGE SET TO:      1,      5

 R-SQUARE =   0.1887     R-SQUARE ADJUSTED =  -0.0818
VARIANCE OF THE ESTIMATE-SIGMA**2 =   5.7333
STANDARD ERROR OF THE ESTIMATE-SIGMA =   2.3944
SUM OF SQUARED ERRORS-SSE=   17.200
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MEAN OF DEPENDENT VARIABLE =   4.4000
LOG OF THE LIKELIHOOD FUNCTION = -10.1834


VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      3 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
W          1.0000      1.197      0.8353     0.465 0.434     0.4344     0.4545
CONSTANT   2.4000      2.623      0.9150     0.428 0.467     0.0000     0.5455

|_ ols q w

REQUIRED MEMORY IS PAR=       1 CURRENT PAR=     781
 OLS ESTIMATION
       5 OBSERVATIONS     DEPENDENT VARIABLE= Q
...NOTE..SAMPLE RANGE SET TO:      1,      5

 R-SQUARE =   0.0865     R-SQUARE ADJUSTED =  -0.2179
VARIANCE OF THE ESTIMATE-SIGMA**2 =   7.9167
STANDARD ERROR OF THE ESTIMATE-SIGMA =   2.8137
SUM OF SQUARED ERRORS-SSE=   23.750
MEAN OF DEPENDENT VARIABLE =   5.0000
LOG OF THE LIKELIHOOD FUNCTION = -10.9901


VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      3 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
W        -0.75000      1.407     -0.5331     0.631-0.294    -0.2942    -0.3000
CONSTANT   6.5000      3.082       2.109     0.126 0.773     0.0000     1.3000
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di)
$$\hat{\gamma}_{12}=\hat{\pi}_{22}/\hat{\pi}_{12}=-0.75/1=-0.75$$
$$\hat{\beta}_{11}=\hat{\pi}_{21}-\hat{\gamma}_{12}\hat{\pi}_{11}=6.5-(-0.75)(2.4)=6.5-1.8=8.3$$

Problem 18.3

a)
sign of β_{11} and β_{21} *are uncertain.*
sign of γ_{12} *is positive* and *less than one.*
sign of β_{22} *is negative.*

b)
$$\pi_{10}=\frac{(\beta_{11}+\beta_{21})}{(1-\gamma_{12})}\quad \pi_{11}=\frac{\beta_{22}}{(1-\gamma_{12})}\quad \pi_{12}=\frac{1}{(1-\gamma_{12})}$$
$$\pi_{20}=\frac{(\beta_{11}-\gamma_{12}\beta_{21})}{(1-\gamma_{12})}\quad \pi_{21}=\frac{(\gamma_{12}\beta_{22})}{(1-\gamma_{12})}\quad \pi_{22}=\frac{\gamma_{12}}{(1-\gamma_{12})}$$
$$\pi_{30}=\beta_{21}\quad \pi_{31}=\beta_{22}\quad \pi_{32}=0$$

c)

Equation`	K-k*	g*-1	Identity
Consumption	2	1	Over
Investment	1	0	Over

$$\gamma_{12}=\pi_{22}/\pi_{12} \text{ or } \gamma_{12}=\pi_{21}/\pi_{11}$$

Since $\beta_{21}=\pi_{30}$ and $\beta_{22}=\pi_{31}$ *are directly estimable*, $\beta_{11}=\pi_{11}(1-\gamma_{12})-\beta_{21}=\pi_{11}(1-\gamma_{12})-\pi_{30}$ or $\beta_{11}=(1-\gamma_{12})\pi_{20}+\gamma_{12}\beta_{21}=(1-\gamma_{12})\pi_{20}+\gamma_{12}\pi_{30}$

d) i) |_ ols y r g

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REQUIRED MEMORY IS PAR=       5 CURRENT PAR=     781
 OLS ESTIMATION
       50 OBSERVATIONS     DEPENDENT VARIABLE= Y
...NOTE..SAMPLE RANGE SET TO:      1,     50

 R-SQUARE =   0.6517     R-SQUARE ADJUSTED =   0.6369
VARIANCE OF THE ESTIMATE-SIGMA**2 =   460.94
STANDARD ERROR OF THE ESTIMATE-SIGMA =   21.470
SUM OF SQUARED ERRORS-SSE=   21664.
MEAN OF DEPENDENT VARIABLE =   86.092
LOG OF THE LIKELIHOOD FUNCTION = -222.732


VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      47 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
R        -0.57292     0.7183     -0.7977     0.429-0.116    -0.0706    -0.0888
G          8.7075     0.9786       8.898     0.000 0.792     0.7878     0.9111
CONSTANT   15.304      14.78       1.036     0.306 0.149     0.0000     0.1778

 |_ ols c r g

REQUIRED MEMORY IS PAR=       5 CURRENT PAR=     781
 OLS ESTIMATION
       50 OBSERVATIONS     DEPENDENT VARIABLE= C
...NOTE..SAMPLE RANGE SET TO:      1,     50

 R-SQUARE =   0.4621     R-SQUARE ADJUSTED =   0.4392
VARIANCE OF THE ESTIMATE-SIGMA**2 =   231.14
STANDARD ERROR OF THE ESTIMATE-SIGMA =   15.203
SUM OF SQUARED ERRORS-SSE=   10864.
MEAN OF DEPENDENT VARIABLE =   52.260
LOG OF THE LIKELIHOOD FUNCTION = -205.476


VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
  NAME    COEFFICIENT   ERROR      47 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
R        -0.19435     0.5086     -0.3821     0.704-0.056    -0.0420    -0.0496
G          4.2116     0.6930       6.078     0.000 0.663     0.6687     0.7259
CONSTANT   16.918      10.46       1.617     0.113 0.230     0.0000     0.3237

|_ ols i r

REQUIRED MEMORY IS PAR=       4 CURRENT PAR=     781
 OLS ESTIMATION
       50 OBSERVATIONS     DEPENDENT VARIABLE= I
...NOTE..SAMPLE RANGE SET TO:      1,     50

 R-SQUARE =   0.0780     R-SQUARE ADJUSTED =   0.0588
VARIANCE OF THE ESTIMATE-SIGMA**2 =   221.07
STANDARD ERROR OF THE ESTIMATE-SIGMA =   14.868
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 SUM OF SQUARED ERRORS-SSE=   10611.
 MEAN OF DEPENDENT VARIABLE =   24.926
 LOG OF THE LIKELIHOOD FUNCTION = -204.889


 VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
   NAME    COEFFICIENT   ERROR      48 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
 R        -0.97483     0.4836      -2.016     0.049-0.279    -0.2794    -0.5220
 CONSTANT   37.938      6.789       5.588     0.000 0.628     0.0000     1.5220
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(ii)

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 |_ ols c y

 REQUIRED MEMORY IS PAR=       4 CURRENT PAR=     781
  OLS ESTIMATION
        50 OBSERVATIONS     DEPENDENT VARIABLE= C
 ...NOTE..SAMPLE RANGE SET TO:      1,     50

  R-SQUARE =   0.8782     R-SQUARE ADJUSTED =   0.8757
 VARIANCE OF THE ESTIMATE-SIGMA**2 =   51.236
 STANDARD ERROR OF THE ESTIMATE-SIGMA =   7.1579
 SUM OF SQUARED ERRORS-SSE=   2459.3
 MEAN OF DEPENDENT VARIABLE =   52.260
 LOG OF THE LIKELIHOOD FUNCTION = -168.337


 VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
   NAME    COEFFICIENT   ERROR      48 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
 Y         0.53401     0.2870E-01   18.61     0.000 0.937     0.9371     0.8797
 CONSTANT   6.2867      2.670       2.354     0.023 0.322     0.0000     0.1203

 |_ ols i r

 REQUIRED MEMORY IS PAR=       4 CURRENT PAR=     781
  OLS ESTIMATION
        50 OBSERVATIONS     DEPENDENT VARIABLE= I
 ...NOTE..SAMPLE RANGE SET TO:      1,     50

  R-SQUARE =   0.0780     R-SQUARE ADJUSTED =   0.0588
 VARIANCE OF THE ESTIMATE-SIGMA**2 =   221.07
 STANDARD ERROR OF THE ESTIMATE-SIGMA =   14.868
 SUM OF SQUARED ERRORS-SSE=   10611.
 MEAN OF DEPENDENT VARIABLE =   24.926
 LOG OF THE LIKELIHOOD FUNCTION = -204.889


 VARIABLE   ESTIMATED  STANDARD   T-RATIO        PARTIAL STANDARDIZED ELASTICITY
   NAME    COEFFICIENT   ERROR      48 DF   P-VALUE CORR. COEFFICIENT  AT MEANS
 R        -0.97483     0.4836      -2.016     0.049-0.279    -0.2794    -0.5220
 CONSTANT   37.938      6.789       5.588     0.000 0.628     0.0000     1.5220
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$$\hat{\gamma}_{12} = \hat{\pi}_{22}/\hat{\pi}_{12} = 4.2116/8.7075 = 0.484$$

(iii) $\hat{\beta}_{11} = \hat{\pi}_{11}(1-\hat{\gamma}_{12}) - \hat{\beta}_{21} = -0.57292\,(1-0.484) - 37.938 = -38.234$

$$\hat{\beta}_{21} = 37.938 \quad \hat{\beta}_{22} = -0.97483$$

The coefficients from the consumption function are over identified and we could have derived a second estimate of each parameter.

(e) Our estimates of the consumption function are consistent, but not necessarily unbiased. The estimates of the investment function are unbiased and consistent since there are no endogenous variables on the right hand side of this equation. It can be estimated consistently and with unbiased estimates using OLS. We have not derived variance estimates for our coefficients and cannot say anything about their reliability.